Problem
Given that the effective half-life of I-131 is 7.5 days, what is the biological half-life of this radionuclide?

Solution
There are several things that must be known to solve this problem, first is the equation that relates $T_{\text{eff}}$, $T_R$, and $T_B$. The next thing that must be known is that $T_R$ for I-131 is 8.05 days. To ensure that a trial and error method is not used, the equation for $T_{\text{eff}}$ must be solved for $T_B$.

$$T_{\text{eff}} = \frac{T_R \times T_B}{T_R + T_B}$$

Multiply both sides of the equation by the quantity of $T_R + T_B$

$$T_{\text{eff}} (T_R + T_B) = T_R T_B$$

Arrange it so $T_B$ is only on one side of the equation.

$$T_{\text{eff}} T_R = T_R T_B - T_{\text{eff}} T_B$$

Factor $T_B$ and isolate it.

$$T_{\text{eff}} T_R = T_B (T_R - T_{\text{eff}})$$

Substitute the given and known values for the effective half-life and the radiological half-life and the answer is straightforward.

$$T_B = \frac{T_{\text{eff}} T_R}{T_R - T_{\text{eff}}} = \frac{(7.5 \times 8.05) \text{days}^2}{(8.05 - 7.5) \text{days}} = \frac{60.3}{0.55} \text{days} = 110 \text{days}$$

It may be useful to convert to the equation that relates the effective decay constant with the radiological decay constant and the biological decay constant.

$$\lambda_{\text{eff}} = \lambda_R + \lambda_B$$

When presented in this way, it is readily apparent which of the processes, radiological or biological, is more important to the overall removal from the body.

Now, for some advice on taking timed, multiple choice exams such as the NRRPT or Part I of the ABHP test. Each question is equally weighted in the exam. The exams are timed such that you have a short, finite amount of time to work on each problem. You may come across a problem such as the one above, and find it tempting to work your way through the problem in order to solve it. Instead skip any problem that you need to use a calculator on until you have completed all the non-calculational problems. This will help in two ways. First, the amount of problems that require a calculator is small, around 10% or less. Next, you will save the battery on your calculator, since you are focused on performing calculations that you have already solved except for the last bit of calculator work. It truly is not worth your precious time to solve problems that require a calculator on your first round through the test. Instead answer all the questions that you know, and set aside the questions that you have to calculate or reason your way through until the end.