Problem Solving in Preparation for the NRRPT® Exam

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Disclaimer

The problems exhibited in this book are similar to the ones on the **NRRPT** examination. However, being able to successfully complete such problems does not guarantee successful results on the examination. Additional study should be performed based upon the references in the application package or as depicted on the **NRRPT** web site. (www.NRRPT.org)

History of Revisions to the Waite Book

Date	Made By	Approved By	Page	Change
2013 Feb 8	D. Tucker	R. Rasmussen	6.2	Corrected problem 6-1. Final answer changed from 18 mR to 180 mR. Reference to answer in discussion corrected and accompanying slight grammatical change (“An” -> “A”). Identified in email from kenneth.j.moran@us.army.mil to NRRPT 2011 December 27.
2013 Feb 8	D. Tucker	R. Rasmussen	6.13	Reinserted Exhibit 6-4 from original. Adjusted pagination of remainder of chapter.
2013 Feb 8	D. Tucker	R. Rasmussen	3.1	Corrected Equation 3-1a per official Errata
2013 Feb 8	D. Tucker	R. Rasmussen	7.12	Corrected equation in step 4 of problem 7-7. Answer unchanged.
2013 Feb 15	D. Tucker	R. Rasmussen	6.2	“Equivalency” sign changed to equal sign in step 5 of Problem 6-1 as identified during review of edits by R. Rasmussen.

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INTRODUCTION

The objective of this book is to guide the reader toward a mastery of working radiation protection problems at the technologist level. It is not a textbook that focuses on establishing an understanding of the physics behind the problems that are presented. Therefore, it is intended primarily for an audience of radiation protection technologists who have a practical understanding of the principles and practice of radiation protection, or health physicists that are starting their refresher study of solving radiation protection problems for certification or other purposes. It is intended to be useful as one of many tools for preparation for the National Registration of Radiation Protection Technologists (**NRRPT**) registration exam.

The following discussion will first outline the learning philosophy that was applied in designing the content and format of the book, then the basic approach that has been adopted to guide users toward mastery of solving radiation protection problems, ending with the organization of the book.

Most books written with objectives similar to that just stated for this book are organized around specific physics concepts and are ordered more or less from the simple to the more complex physical principles. Unfortunately, this requires the reader to be constantly shifting from one mathematical level to another, and for the reader who has more confidence in his/her ability to understand the radiation protection principle involved than in the mathematical manipulations required to solve the problem, this introduces unnecessary distractions into the learning process.

The learning philosophy that serves as the cornerstone of this book involves five steps:

- offer the problem solving tools
- show how to use the tools by a simple, but complete, walk through of examples
- cover all eventualities in exercise problems, maximizing the utility of each important equation by solving it for all of its variables as unknowns and by solving for variables for which existing variables are surrogates and for which multiple stage solutions are required.
- test comprehension
- provide opportunities for feedback from the process in areas of weakness

The approach that has been used to develop the organization of the book is based on the concept that to pursue mastery of the material one must:

- build the necessary mathematical skills to work the radiation protection problems
- present the specific mathematical techniques necessary to solve the relevant equations
- reinforce the techniques by walking through a few simple example mathematical problems
- master the techniques by working radiation protection problems

- master radiation protection problem solving by working more problems with less guidance

To apply these concepts and to achieve these objectives, the book has been organized as follows. The first chapter discusses the technique that is recommended to be followed to solve any radiation protection problem, no matter what kind or at what mathematical level. All example problems are offered in the format of this technique and, where fully developed answers are given to exercise problems, they are given in this format. Where only the answers are given to exercise problems, it is recommended that the reader use the same approach.

Beyond the problem solving technique chapter, the book is organized into parts. Each part contains radiation protection problems that apply only one level of mathematical manipulation. The parts advance from the use of Algebra, to Logarithms and Exponentials, to Statistics, and finally Trigonometry. Each part begins with a mathematics chapter that describes the mathematical tools required to solve the radiation protection problems presented in the subsequent chapters of that part.

Each part is divided into Chapters that apply the mathematical techniques of that part to specific types of radiation protection problems. For instance, the first chapter in the Logarithms and Exponentials part is "*Radioactivity and Radioactive Decay*". The number of radiation protection chapters within each part is variable, being determined by the number needed to cover the radiation protection concepts important to achieving the stated objective of the book.

The text of each chapter contains example problems with full solutions, developed around the recommended solution technique presented in Chapter 1. At the end of the book, there are multiple appendices that give information that otherwise would have to be drawn from other sources before problems could be solved. For instance, one appendix offers commonly used conversions factors. Another gives formulas and equations that are used throughout the book in problem solving.

Based on this organizational format and content, it is recommended that no matter what intent motivates the reader to use the book, even if it is to learn how to solve a specific type of radiation protection problem that is contained in a specific chapter in the book, he/she first consult the solution technique chapter, then the appropriate mathematics chapter before going to the specific problem area of interest. Once the tools and skills necessary to successfully solve a particular type of radiation protection problem are at hand, then one can enter the system at the level most appropriate to the existing level of mastery.

CHAPTER 1

TECHNIQUE FOR SOLVING RADIATION PROTECTION PROBLEMS

This chapter discusses the technique that is recommended to be followed to solve any radiation protection problem, no matter what kind, or at what mathematical level. All example problems are offered in this format and, where fully developed answers are given to exercise problems, they are also given in this format. Following this brief introduction to the topic, each component of the problem solving technique is presented and discussed in detail. Then the various components are combined into an integrated process that can always be followed. Application of the integrated process and presentation format is then demonstrated for an example problem.

Any activity for which consistency is important to success, like problem solving, making a good golf shot or delivering a good presentation; proper planning, preparation and execution are of paramount importance. As the golfing situation would indicate, no one way is the only way, but each individual should have a consistent and preferred way. The problem solving technique discussed in this chapter is such a way. It has been proven to work over the years, and will serve as a model if individual problem solvers elect to adopt a different technique. The important thing is to select as early in this book as possible a technique, probably including the same basic steps as the recommended one, to apply to every problem, so that it can be engrained in the problem solver's approach before many of the more complex and difficult problem types occur.

The technique includes the following steps, in order:

- read the problem and select the appropriate mathematical relationship among the variables involved and isolate the unknown by treating each side of the equation identically, as described in the appropriate mathematics chapter
- simplify the equation
- validate the problem setup using unit analysis
- plug in the values for known quantities, as given in the problem, established constants/conversion factors and results of side-bar calculations
- quantify the unknown
- do a reality check of the solution.

Read the Problem, Select the Proper Equation, and Isolate the Unknown

Step number one of the process is often an iterative process, especially early in the process of a problem solver becoming comfortable with word-type problems. The common scenario is that the problem will be read without the reader assimilating the information being given in the context of the appropriate variables and variable relationships. The minimum goal of the first few readings of the problem should be to identify what kind of problem is being presented. Hopefully, by the end of this book the reader will be convinced that there are only about two dozen types of problems that typically dominate radiation protection practice. Furthermore, not many more than the two dozen equations must be mastered before these types of problems can be easily characterized, understood and solved.

So, after the first reading or so, it should be recognized that the problem, for example, involves converting current in a radiation detector to a radiation exposure. Knowing the relationship between these variables, and the other information that is crucial to the solution of such a problem, one can read the problem with a much greater understanding when looking for the specific information that will enable the appropriate equation to be solved.

Any variable in the relationship identified early in this step of the process may be the unknown variable. In some cases it may be the obvious one, like the exposure rate in the hypothetical problem just discussed. More likely, some other variable will be the unknown, like the pressure of the gas in the detector, or the volume of the detector. In each case, the same first steps are required. The details of this step in the process will differ, depending on the type of equation and the variable that is the unknown. As was discussed in the introduction to this book, the problems are organized in such a way that the reader can assume that the approach to this step falls within the purview of the mathematics chapter that starts the part. Therefore, if the radiation protection problem appears within the Algebra part, an algebraic approach to isolating the unknown variable will be appropriate, and those techniques will have been discussed in the first chapter in that part.

Simplify the Equation

In many cases, especially in the more complex problem types, when the unknown variable is isolated, the resulting equation is messy, in that it may contain the same variable in several different places, or different mathematical operators may be strewn throughout the equation. To enhance the possibility that the equation makes physical sense, it is a good policy to simplify the equation. Then it should be obvious that two variables are directly proportional or inversely proportional, and the unit (or dimensional) analysis step in the process will be more understandable and more straightforward.

Validate the Problem Setup

It is of little value to proceed with solving the problem numerically if the problem is set up incorrectly. A quick way to establish some certainty that the setup is correct is to analyze the units, as they reside in the simplified equation and see if the units on both sides of the equation are the same, or can be converted to be the same, and if the answer comes out in the unit it should. This step emphasizes the importance of carrying along the units of each variable when solving radiation protection problems. Simply having numbers for each variable is not enough to ensure a correct solution of radiation protection problems. We use many units that, when used incorrectly, can result in meaningless solutions, or, at best, solutions that are many orders of magnitude in error.

There are two situations where this step takes on a slightly different approach. The first of these is when empirical formulas are involved. In many of these cases in radiation protection, relationships have only been experimentally observed to describe the right relationship between several variables, without regard to their units. In these cases, unit analysis is not useful.

The second case is where multiplicative models are involved. In these cases, the units drive the whole process and the numbers are dragged along with the units, instead of the usual dragging the units along with the numbers. In other words, the whole process of solving these types of problems involves unit analysis, in contrast to having unit analysis only one late step in of the process.

An example of this step can be demonstrated using the following radiation protection problem:
A worker is exposed to a radiation field of 0.5 mR/hr exposure rate (ER). The exposure lasts for 4 hours exposure time. (ET)

What is the worker's total exposure (TE)?

Step 1: Isolate the unknown variable
ER • ET = TE

Step 2: Simplify the equation

(not needed since the equation is already in a form to solve for the unknown called for in the problem)

Step 3: Validate the problem setup
[mR/hr] • [hr] = [mR]

In this case the answer is in the desired unit and with the cancellation of the [hr] and [1/hr], the units on both sides of the equation are the same, indicating that no conversions, etc. are required. This would not be the case if the exposure rate was in terms of [R/hr] and the desired answer for (TE) was in [mR].

Plug in Values for Known Quantities, Constants and Conversion Factors

Now with some certainty in the problem setup established, one is left to the actual solving of the problem for the answer. For the example problem given in the discussion of step three of the process, this step looks like:

Step 4: 0.5 [mR/hr] • 4 [hr] = 2 [mR]

The use of constants or conversion factors is not needed for the example problem being discussed, because all of the variables are either the unknown or are given values in the problem. However, as pointed out before, if the exposure rate were given in terms of [R/hr] and the (TE) needed to be stated in terms of [mR] then this step would involve the conversion of the [R/hr] to [mR/hr]. This, of course would be accomplished by multiplying the [R/hr] value by 1000. The need to do this operation as part of the solution would have been discovered in Step 3, because the units would not have come out in the unit analysis. So, the new Step 3, for the case where (ER) is given in [R/hr] goes as follows:

[R/hr] • [mR/R] • [hr] = [mR]

Again all of the conditions for validity of the problem setup are met. Without the conversion factor from [R] to [mR] this is not so.

The characteristic that differentiates a constant or conversion factor from a variable in the equation is that neither constants nor conversion factors are usually given with the problem. It is expected that these values are either known or can be retrieved from a reference. The constants and conversion factors most commonly required to solve problems in this book are given in the appendices, as described in the introduction to the book. The second characteristic, primarily of conversion factors, is that they may not even appear as variables in the equation. For example, in the example problem where exposure rate is in terms of [R/hr], there is no indication in the equation identified in step 1 of a conversion being needed. One must recognize that [R] and [mR] are different and that the difference is a factor of 1000 mR/R. This reinforces the need for the dimensional (or unit) analysis in Step 3.

Quantify the Unknown

Now that all of the needed information for solving the equation is at hand, it remains only to "turn the crank." In the original example problem this step is:

Step 5: TE = 2mR

Conduct Reality Check

One should not believe the result of every calculation on the basis of faith. Sometimes values are entered incorrectly into computers or calculators, sometimes operations, other than those intended, are executed unknowingly, etc. It is up to the problem solver to identify the more gross of these types of errors. One way is to conduct a reality check. This means to explicitly ask yourself if the answer makes sense. If your intuition tells you that the answer should be a few mR, and your answer indicates hundreds of mR, perhaps it would be prudent to look through the solution process to either validate the answer or identify where the error occurred.

Other methods available to be used in reality checks are:

- independent order of magnitude estimate
- rule of thumb estimate

- graphic representation estimate
- use of nomographic data.

An example problem that illustrates the point is:

A point source of Cs-137 gives 0.33 R/hr (ER_1) per Ci at 1 meter (D_1).

What is the exposure rate (ER_2) at 5 meters?

Step 1: Isolate the unknown variable:

$$ER_2/ER_1\ [R/hr \div R/hr] = D_1{}^2/D_2{}^2\ [m^2 / m^2]$$

$$(ER_2/ER_1) \bullet ER_1 = (D_1^2/D_2^2) \bullet ER_1$$

$$ER_2 = (D_1^2/D_2^2) \bullet ER_1$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$[R/hr] = [m^2] \bullet [R/hr] / [m^2]$$

Step 4: Plug in known quantities:

$$ER_2 = (1^2 \bullet 0.33)/5^2$$

Step 5: Solve for unknown:

$$ER_2 = 0.013\ R/hr$$

Step 6: Conduct reality check:

In doing a reality check of this answer, one would probably look at this exposure rate at 5 meters compared with the exposure rate at 1 meter, to make sure that the 5 meter rate is lower. Sometimes, like when the distance is doubled, the math can be done in your head and even the numerical value of the answer can be validated. In this case maybe this is not possible, but if the 5 meter exposure rate is MORE than the 1 meter rate, we know something is wrong; perhaps the 1 and 2 subscripts were exchanged on one side of the equation or the other. (It is interesting to note that such a mistake would not be picked up by the dimensional analysis since both the denominator and the numerator have the same units.

Where the distances in such a problem are much different, often the units of the exposure rates will need to be addressed to make the numerical values more understandable and useful. For instance, 1/1000 of an R/hr is more useful when using an instrument if it is expressed in mR/hr. In such cases formal conversion of units is necessary. Casual unit conversion often leads to disastrous results, especially if done in the field.

As discussed earlier, it is this problem solving technique that is recommended to be applied to all problems in this book. The problems have been set up in this way to try to lead by example. Perhaps you can begin to see how this approach is designed to catch mistakes before the solution gets out of the problem solver's hands. Further confidence in its ability to keep our mistakes to a minimum and to ourselves, as much as possible, will develop from its use on the problems that follow in the subsequent chapters.

CHAPTER 2

ALGEBRA

Most of the mathematics used to solve radiation protection problems is covered under the general area called Algebra. Algebra deals with the manipulation of real numbers. To a radiation protection specialist, the most important subset of manipulations is the solving of algebraic equations with unknowns. This section provides a brief summary of algebraic laws and manipulating algebraic equations.

Algebraic Equations

The algebraic equation is the most important concept used in solving mathematical problems. An equation is a statement asserting the equality of two expressions that are separated into left and right sides and joined by an equal sign. For example, $5 + 7 = 12$. An equation can also be written with one or more variables, or unknowns. The equation $x + 7 = 12$ is an equality only when the variable $x = 5$. The value 5 is the solution of this equation. The numbers 7 and 12 are constants in the equation. For the equation $x + y = 12$, both x and y are variables and there is an infinite number of solutions or combinations for x and y, *e.g.*, $x = 1$, $y = 11$; $x = 5.5$, $y = 6.5$; $x = -5$, $y = 17$, *et cetera*. The end product of algebra is solving mathematical equations.

In radiation protection, the concept of equal physical quantities is encountered routinely. An equation is merely the statement of this equality. For example, the dose equivalent (H) is equal to the product of absorbed dose (D), a quality factor (QF), and a distribution factor (n). In algebraic terms, $H = D \bullet QF \bullet n$. This equation becomes interesting as a problem to be solved when one of the expressions, such as dose equivalent, is an unknown.

Solving Algebraic Equations

One step in solving a radiation protection problem is to express the known and unknown quantities in terms of an algebraic equation. Then, this equation can be simplified and solved. To solve an algebraic equation, a three-step process is used. Here we should note how these three steps fit into the six-step technique we are recommending for setup, solution and verification of a problem. Note that this process deals with equations with a single unknown. Equations with multiple unknowns are discussed later, in *Solving Systems of Equations*.

Step 1: Isolate the unknowns, or variables, on one side of the equation and the constants on the other side of the equation.

Step 2: Simplify the equation to the form $x = C$, where x is the unknown and C is a constant. C is then the solution to the unknown.

Step 3: Verify the solution by putting the solution for x into the *original* equation to check to see if the equality is correct.

For example, solve $4x + 5 + 2x = 29$

Step 1: Isolate the unknowns, or variables, on one side of the equation and the constants on the other side of the equation.

$4x + 5 + 2x = 29$ is the same as $4x + 2x + 5 = 29$. (This is the cumulative property of addition, which states that you can add numbers in any order you want). By subtracting 5 from each side of the equation, the expressions become: $4x + 2x = 24$.

Step 2: Simplify the equation to the form $x = C$, where x is the unknown and C is a constant. C is then the solution to the unknown.

$4x + 2x = 24$. Simplifying the left side of the equation yields $6x = 24$. By dividing each side by 6, the equation becomes $x = 4$.

Step 3: Verify the solution by putting the solution for x into the *original* equation to check to see if the equality is correct.

To verify, $4\ (4) + 5 + 2\ (4) = 29$. $16 + 5 + 8 = 29$ ✓

Two important concepts about the above example: first, when you isolate the variable and simplify the expression, you must perform the same operation on both sides of the equations. If you subtract a number from the left side of the equation, you must subtract the same number from the right side of the equation. If you multiply the left side of the equation by a number, you must multiply the right side by the same number. By manipulating both sides of the equation the same way, you keep the two sides equal. For multiplicative model use, it should also be mentioned that multiplication by 1 could be done as many times as required by a specific problem. Also it should be mentioned that many times it may not appear that a quantity is equal to 1, like 24 hours per day, 35 eV per ion pair, etc.

Secondly, there is a set of laws that govern some manipulations of the expressions. An example is the cumulative property of addition, illustrated above. These laws, or properties, a summarized below:

Algebraic Laws

Commutative Law of Addition $a + b = b + a$

Commutative Law of Multiplication $a(b) = b(a)$

Associative Law of Addition $a + (b + c) = (a + b) + c$

Associative Law of Multiplication $a(bc) = (ab)c$

Distributive Law $a(b + c) = ab + ac$

For a practical example of solving an algebraic equation with one unknown, let's return to our expression for dose equivalent. As we said before, the dose equivalent (H) is equal to the product of absorbed dose (D), a quality factor (QF), and a distribution factor (n). In algebraic terms, H = D • QF • n. Now, let's determine what absorbed dose (D) of alpha radiation to the lungs would yield a dose equivalent of 5 rem. The quality factor (QF) for alpha radiation is 20 rem per rad and the distribution factor (n) is 1. The solution is:

Step 1: Isolate the unknowns, or variables, on one side of the equation and the constants on the other side of the equation.

The equation, in symbolic terms, is H = D • QF • n. The problem tells us the values of H, QF, and n. So, 5 rem = D • 20 rem/rad • 1. To isolate the unknown, D, on the right side of the equation, divide both sides by 20 rem/rad. Then, the expression is:

$$\frac{5\text{ rem}}{20\text{ rem/rad}} = \text{D rad}$$

Step 2: Simplify the equation to the form $x = C$, where x is the unknown and C is a constant. C is then the solution to the unknown.

0.25 rad = D

Step 3: Verify the solution by putting the solution for x into the *original* equation to check to see if the equality is correct.

5 rem = 0.25 rad • 20 rem/rad • 1. This equation reduces to 5 rem = 5 rem✓.

Note that in this practical example, the constants and variables had units associated with them. It is important to keep the units with the constant or variable and make sure the units cancel out. In the example above, the product of the units *rad* and *rem per rad* equaled *rem*, since the unit rad canceled out, *i.e.*, the unit *rad* was in the numerator and denominator of the expression. Later chapters with discuss manipulating units in more detail, as well as review the units *rem* and *rad*.

Solving Systems of Equations

To solve algebraic equations for unknowns, the number of unknowns must be equal to or less than the number of equations you have to work with. When you have two unknowns, you must have at least two equations with that unknown. When you have two or more equations with unknowns, you have a system of equations. The example below illustrates a system of equations.

$$5x + 2y = -19$$
$$2x + y = 4$$

To solve the system of equations, you need to follow these steps:

Step 1: Reduce the system to an expression with one unknown. You do this by solving one of the equations for one of the variables.

Step 2: Substitute the solution for the one variable into the other equation. This substitution should give you one equation with one variable.

Step 3: Solve the single equation for the single unknown using the method described in *Solving Algebraic Equations*, above.

Step 4: Substitute the solution from Step 3 into either of the two equations and solve for the other variable using the method described in *Solving Algebraic Equations*, above.

Step 5: Verify the solution by putting the solution for the unknowns into both *original* equations to check to see if the equalities are correct.

In our example above:

$$5x + 2y = -19$$
$$2x + y = 4$$

Step 1: Reduce the system to an expression with one unknown. You do this by solving one of the equations for one of the variables.

Subtracting 2x from each side of the second equation yields $y = 4 - 2x$.

Step 2: Substitute the solution for the one variable into the other equation. This substitution should give you one equation with one variable.

In the first equation, we substitute $4 - 2x$ for y, or $5x + 2(4 - 2x) = -19$. This equation is the same as $5x + 8 - 4x = -19$ when we multiply the value 2 through the parentheses. This is the same as $5x - 4x + 8 = -19$.

Step 3: Solve the single equation for the single unknown using the method described in *Solving Algebraic Equations*, above.

First, we simplify the expression to $1x + 8 = -19$. Then, we subtract 8 from both sides to get $1x = -27$, or $x = -27$. To check, we have $5 (-27) + 2 [4 - 2(-27)] = -19$. $-135 + 2 [4 - (-54)] = -19$. $-135 + 2 (58) = -19$. $-135 + 116 = -19$. $-19 = -19$✓

Step 4: Substitute the solution from Step 3 into either of the two equations and solve for the other variable using the method described in *Solving Algebraic Equations*, above.

Substituting -27 for x in the second equation, we get $2 (-27) + y = 4$. $-54 + y = 4$. If we add 54 to each side, we get $y = 58$. Checking the equation, we get $2 (-27) + 58 = 4$. $-54 + 58 = 4$. $4 = 4$✓

Step 5: Verify the solution by putting the solution for the unknowns into both *original* equations to check to see if the equalities are correct.

Checking in the first equation, $5 (-27) + 2 (58) = -19$. $-135 +116 = -19$. $-19 = -19$✓. Checking the second equation, we get $2 (-27) + 58 = 4$. $-54 + 58 = 4$. $4 = 4$✓.

If your system has more than two equations, then you repeat Step 1, continually reducing your system until you can isolate a single unknown.

Polynomials

Most of the equations that you will deal with in solving radiation protection problems and the equations illustrated above are linear equations. In linear equations, the variable is not raised to a power. A polynomial is an equation where a variable is raised to a power.

A quadratic equation is an equation containing the second power (square) of an unknown but no higher power. The equation $2x^2 - 32 = 0$ is a quadratic equation. A quadratic equation has two roots, both of which satisfy the equation.

Solving Quadratic Equations

There are different techniques used to solve quadratic equations, based on the form of the equation. The only technique discussed here is solving a pure quadratic using the square root method. This method is the only method you should need when solving radiation protection problems. There are four steps in solving pure quadratic equations by taking the square root.

Step 1: Isolate the x^2 term on one side of the equation.

Step 2: Simplify the expression to the form $x^2 = C$.

Step 3: Take the square root of both sides of the equation.

Step 4: Check the roots.

Using our example above, $2x^2 - 32 = 0$:

Step 1: Isolate the x^2 term on one side of the equation.

Add 32 to both sides of the equation, $2x^2 = 32$.

Step 2: Simplify the expression to the form $x^2 = C$.

Divide both sides of the equation by 2, $x^2 = 16$.

Step 3: Take the square root of both sides of the equation.

The square root of x^2 is x. The square root of 16 is +4 *and* -4.

Step 4: Check the roots.

$2(4)^2 - 32 = 0$. $2(16) - 32 = 0$. $32 - 32 = 0$. $0 = 0$✓
$2(-4)^2 - 32 = 0$. $2(16) - 32 = 0$. $32 - 32 = 0$. $0 = 0$✓

Note that there are two solutions. Both +4 and -4 squared equals 16. However, when solving physical problems, only one solution may make sense, *i.e.*, negative values may not make sense from some quantities. In these cases, you must perform a sanity check on the solution to determine the correct sign of the answer.

Generally, when dealing with square roots, the answers come out to whole numbers in math books only. You may need a calculator to estimate the value of a square root. For example, if we change our equation to be $x^2 - 32 = 0$, then the solution is:

Step 1: Isolate the x^2 term on one side of the equation.

Add 32 to both sides of the equation, $x^2 = 32$.

Step 2: Simplify the expression to the form $x^2 = C$.

The equation is already simplified, $x^2 = 32$.

Step 3: Take the square root of both sides of the equation.

The square root of x^2 is x. The square root of 32 is +6.656842... *and* -6.656842.... These results are from a calculator (the calculator provided only the positive result).

Step 4: Check the roots.

$(6.656842)^2 - 32 = 0$. $32 - 32 = 0$. $0 = 0$✓
$(-6.656842)^2 - 32 = 0$. $32 - 32 = 0$. $0 = 0$✓

Graphical Representation of Equations

It may be useful to plot an equation on a graph to determine solutions to a problem. Linear equations result in straight lines when plotted on a graph, while polynomials are represented by curves. The graph of an equation with two variables is the set of all points whose coordinates satisfy the equation.

In Chapter 4, we will review that the fractional amount of monoenergetic electron energy incident on a target that is converted to Bremsstrahlung x-rays, F, is given by the following equation:

$$F = 3.5 \times 10^{-4} \bullet Z \bullet E$$

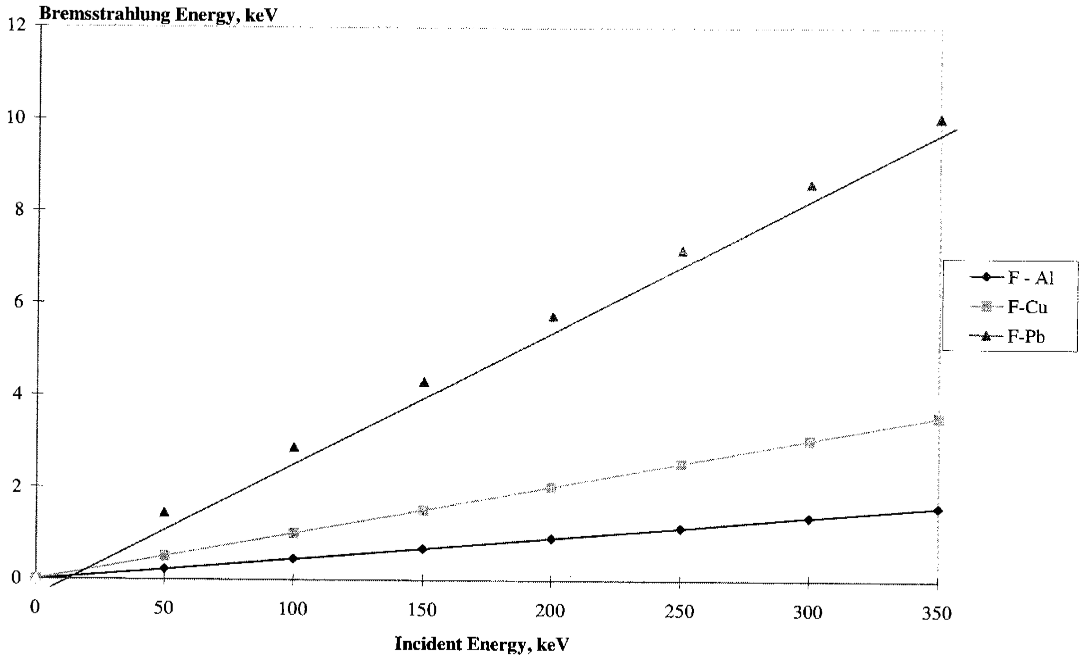
Z is the atomic number of the target element and E is the energy of the monoenergetic electrons. If the target is aluminum, atomic number of 13, the equation is:

$$F = 3.5 \times 10^{-4} \bullet 13 \bullet E, \text{ or } F = 0.00455 \bullet E$$

The following exhibit gives values for F for given values of E for aluminum, copper (Z = 29), and lead (Z = 82):

Exhibit 2-1 below presents a graph of the equation.

Exhibit 2-1.
Bremsstrahlung Energy



Linear equations have the form y = mx + b. The constant "m" is the slope of the line and the constant "b" is the y-intercept - the point the line crosses the y-axis. In the example above, the slope is $Z \bullet 3.5 \times 10^{-4}$ and the y-intercept is 0. From Exhibit 2-1, it can be seen that the higher the Z, the greater the slope of the line.

Power Functions

Mathematical expressions for exponential growth or decay have many applications in radiation protection. Many of these applications are expressed as a power of *e*, the base of natural logarithms. These applications are discussed in Part 2 of this book. However, other applications, including the attenuation of electromagnetic radiation, deal with powers of 2 and 10. The section below discusses the power functions of 2 and 10.

A power function is expressed as follows:

$$y = x^n$$

For the presentation here, we will limit x to the values of 2 or 10, but the function is defined for all non-zero values of x. The example below illustrates a classic example of exponential growth.

One day, a child is given a dollar and puts it in her piggy bank. In each subsequent day, the child puts in twice as much money as she did the day before. So, on the next day (Day 1), she puts in 2 dollars, on Day 2 she puts in 4 dollars, on Day 3 she puts in 8 dollars, and so on. How much money must she put in the bank on the tenth day? The twentieth day?

To solve the example, we use the power function. In this example, x is 2. When you set n to the number of days after the child was given the dollar, then y is the number of dollars she must put in her piggy-back on that day.

In Day 10:

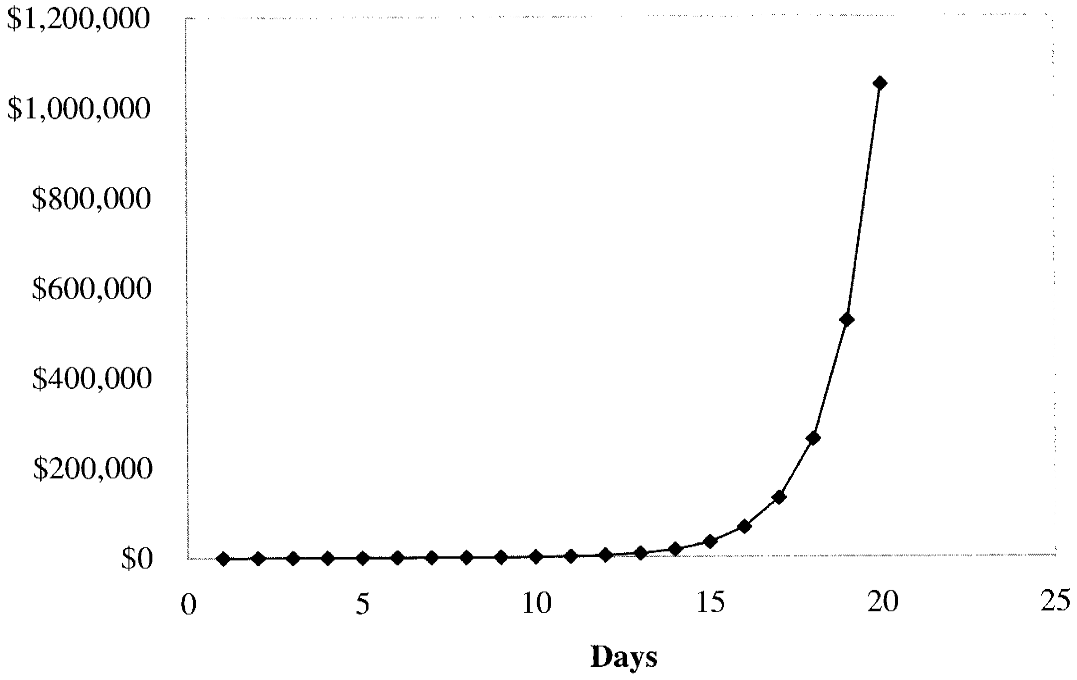
$y = 2^{10}$, which is \$1,024.

In Day 20:

$y = 2^{20}$, which is \$1,048,576

As we can see from the numbers above, there is a rapid increase in the values of y. Exhibit 2-2 illustrates the exponential growth in our example.

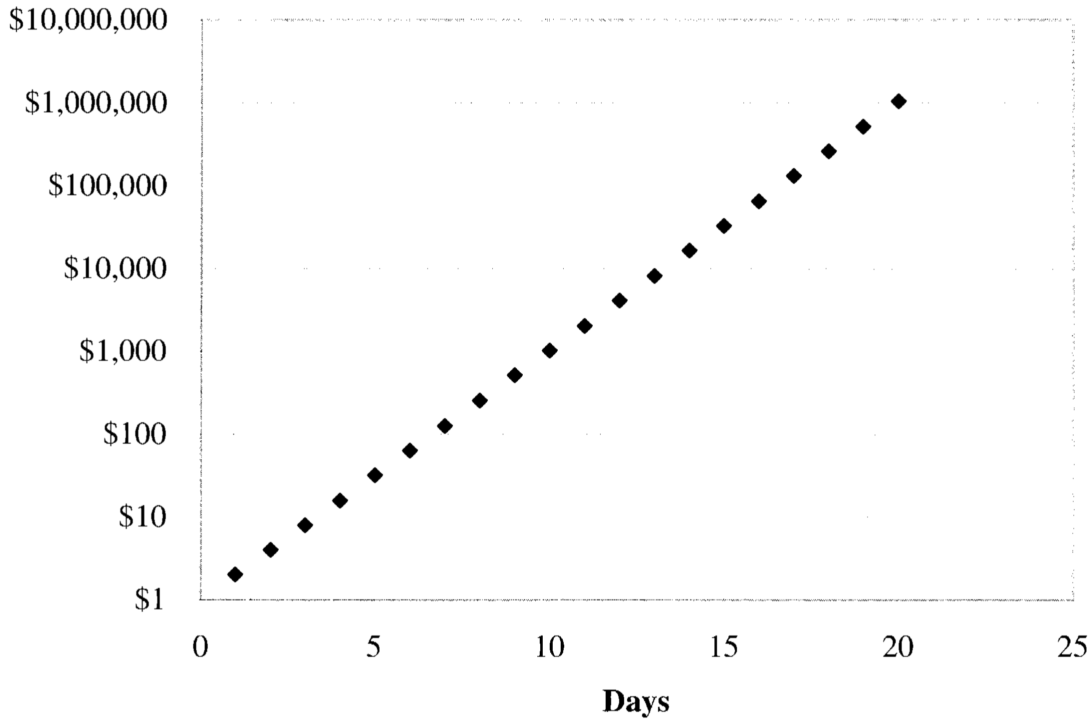
Exhibit 2-2.
Power of 2



It can be seen from the figure above that for graphs capturing exponential growth, details can be lost because of the wide range of values on the y-axis. To see the detail in Exhibit 2-2 from Day 1 through 15, you could break the graph into two parts, one illustrating the growth from Day 1 through Day 15 and the other the growth from Day 15 through Day 20. A more common way to capture the detail in the graph would be to change the y-axis scale to logarithmic. With a logarithmic scale, values of 10 are evenly spaced up the y-axis (1, 10, 100, 1000, 10000, and so on). Exhibit 2-3 presents the same function as Exhibit 2-2, however the y-axis is a logarithmic scale. Since the x-axis is still linear, this plot is called a semi-log plot. If both axes were logarithmic scales, then the plot would be a log-log plot. As can be seen from the plot, the sharp upwards curved plot turns into a straight line in a semi-log plot.

A more appropriate example of the power function as it relates to radiation protection is exponential decay. In Chapter 4, we will discuss half-value layers (HVL) and tenth-value layers (TVL), which are the thickness of a shield required to reduce an exposure rate to one-half or one-tenth its original value. The reduction in the exposure rate can be expressed as a power function of either 2 (when talking about HVLs) or 10 (when talking about TVLs). The example below illustrates the application.

Exhibit 2-3.
Power of 2, Semi-Log Plot



Three inches of a particular shield reduces the dose rate from a point source of cesium-137 by one-half (*i.e.*, the HVL for the shield for Cs-137 is three inches). By what factor would the exposure rate be reduced if 10 inches of shielding were applied?

1 HVL = 3 inches.

10 inches/3 inches per HVL = 3.33 HVLs in 10 inches of shielding.

$y = 2^{-3.33} = 0.10$.

Therefore, the exposure rate would be reduced to one-tenth its original level by adding 3.33 HVLs, or 10 inches, of shielding. For this example, 10 inches is the TVL for the shield, since the dose rate was reduced by a factor of 10. Note that the exponential term in the power function is negative, representing an exponential decay.

Exhibit 2-4 shows the plot of dose rate from a point source of Cs-137, as a fraction of the original, versus the number of HVLs. Exhibit 2-5 shows the same plot in semi-log form. Again, the semi-log plot shows a straight-line plot.

Exhibit 2-4.
Dose Rate With Increasing Shielding

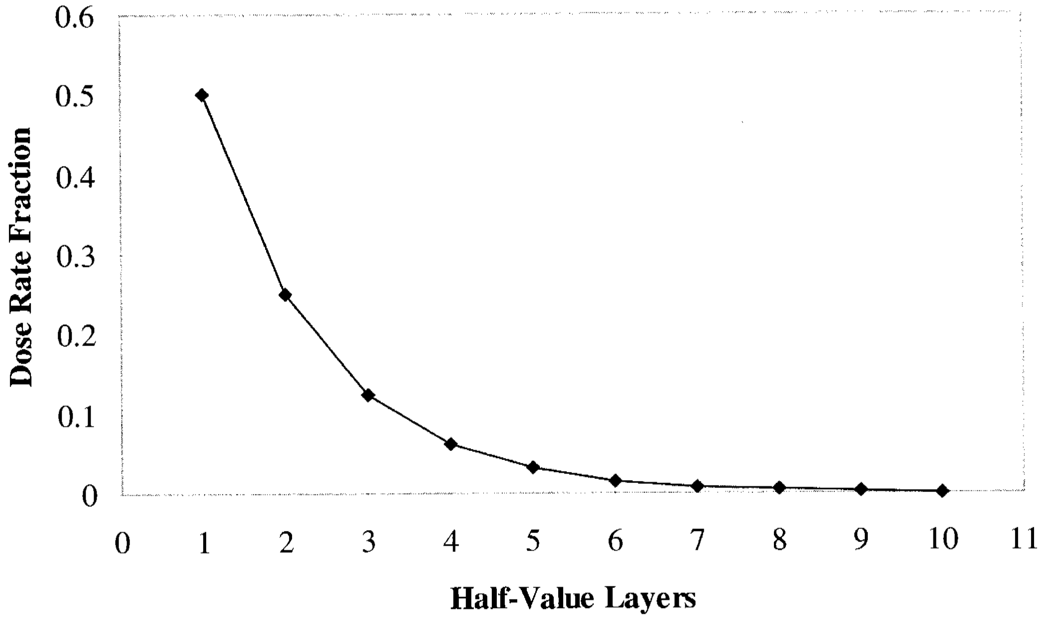
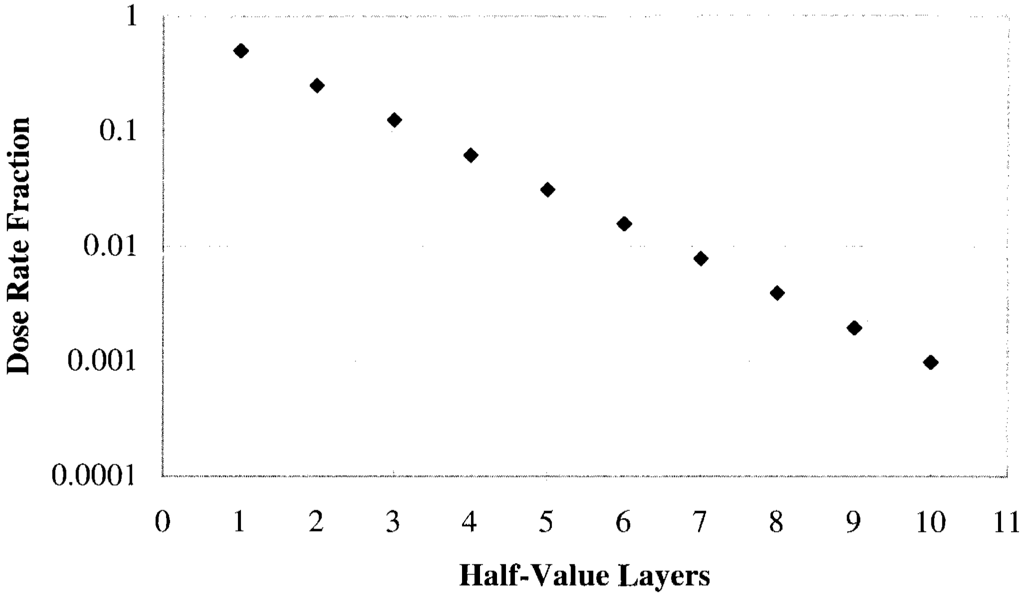


Exhibit 2-5.
Dose Rate Decrease, Semi-Log Plot



CHAPTER 3

PARTICULATE RANGE BEHAVIOR

An important component of becoming competent in solving range/energy equations is that of dispelling the idea that all range/energy problems are the same; that is to say, given a type of radiation and an energy, what is the range? In this chapter it will be demonstrated that there are 5 or 6 ways that this type of problem can come up, only one of which is the given a type of radiation and energy, what is the range? The several ways will be illustrated in the example problems in this chapter, and the demonstration will begin in the context of alpha particles.

Alpha Particles

All of the alpha range problems presented in this chapter are doable through the use of algebra, which was summarized in the previous chapter. However, before we get to the solving of problems, perhaps a brief discussion of the physics of alpha interactions might be useful.

Alpha particles are emitted from heavy radioactive nuclei monoenergetically and exhibit range behavior. That is, all alpha particles emitted with the same energy will travel about the same distance in a medium.

Alpha particles are made up of two protons and two neutrons and, therefore have a mass number of 4 and a charge of +2. The large mass, relative to surrounding electrons, and the associated relatively slow velocity, in addition to the strong attractive force of the alpha particle for surrounding electrons, make the alpha interaction with matter one of the alpha causing ion pairs along its track and eventually becoming neutralized by capturing electrons. If we plot the number of alphas along a track versus the absorber thickness, a curve like that shown in Exhibit 3-1 results. Where the number of alphas goes to zero is the distance referred to as the “range”. It is this quantity that is the focus of problems in this chapter.

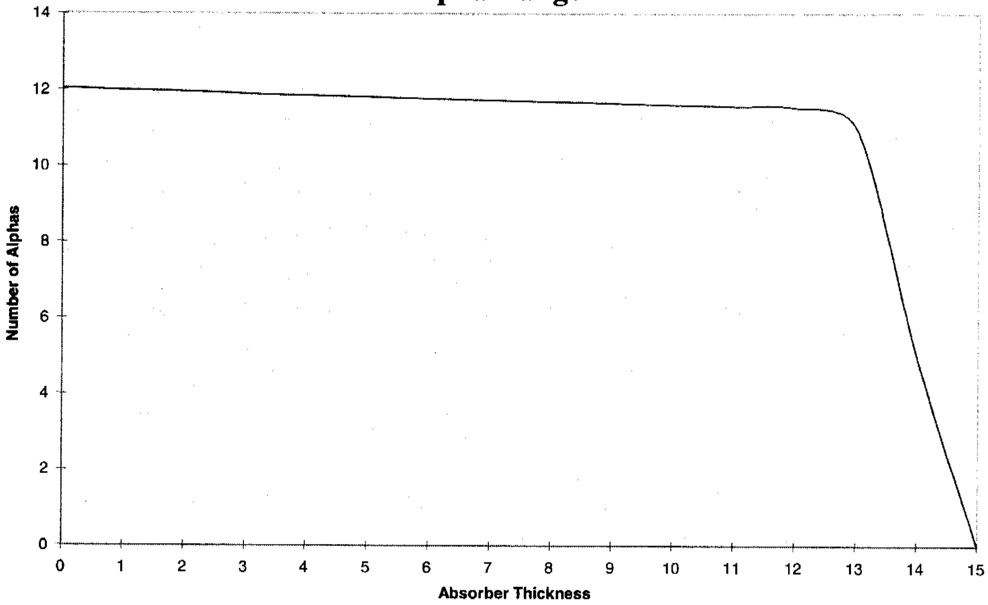
Because each ion pair created by the passage of the alpha particle requires the same amount of energy (about 35 eV), it is observed that the more initial energy given the alpha particle, the longer the range. The question before us, then, is what is the mathematical relationship between the range and the energy of alpha particles?

For air as the absorbing medium, the empirical relationships are:

$$R_a[\text{cm}] = 0.56E \quad \text{for } E<4 \text{ MeV} \qquad \text{Equation 3-1a}$$

$$R_a[\text{cm}] = 1.24E - 2.62 \quad \text{for } 4<E<8 \text{ MeV} \qquad \text{Equation 3-1b}$$

Exhibit 3-1.
Alpha Range



where,

R_a = the range of the alpha in air
E = the energy of the alpha [MeV]

It should be noticed that both of the above equations are of the form of a straight line, y = mx + b. Therefore, they can be plotted on linear graph paper, as they are seen in Exhibit 3-2 to yield joined straight lines which are useful in solving alpha particle range/energy problems where the precision available from purely mathematical approaches is not necessary. An even cruder method of estimating the range of an alpha with energy below 4 MeV is by the rule of thumb derived from the indication in Equation 3-1a that the range in [cm] is approximately one half the alpha particle energy in [MeV].

Taking the process one step further, we need to address the fact that many times we are not interested in the range of the alpha in air, but in some other material. For these situations, the range in any substance is given by:

$$R_m\ [\mathrm{mg/cm^3}] = \{(0.56)\ (A_m^{1/3}) \bullet R_a\ [\mathrm{cm}]\} \qquad \text{Equation 3-2}$$

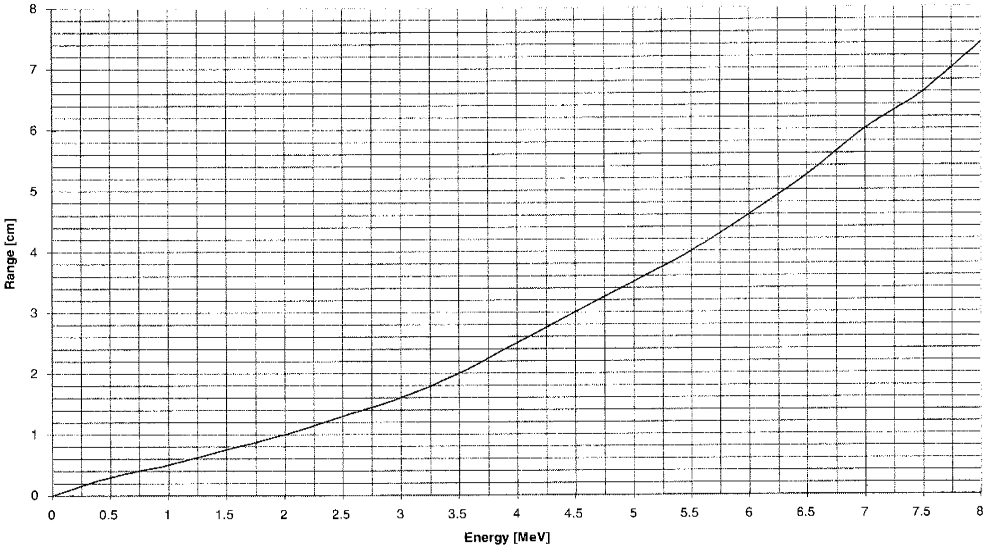
where,

R_m = the range of the alpha in the medium

A_m = the atomic mass number of the medium

R_a = the range of the alpha in air

Exhibit 3-2.
Alpha Range vs. Energy



The calculation of effective atomic mass number is done through the relationship:

$$A_m = (n_1A_1+n_2A_2 +...+n_nA_n)/\{(n_1)(A_1^{1/2}) + (n_2)(A_2^{1/2}) +...+(n_n)(A_n^{1/2})\} \quad \text{Equation 3-3}$$

where,

$n_1...n_n$ = the number of each atom per cm^3

$A_1...A_n$ = the atomic mass numbers of each atom

For tissue as the absorbing medium, the relationship simplifies to:

$$R_a\rho_a = R_t\rho_t \quad \text{Equation 3-4}$$

where,

R_a = the range of the alpha in air [cm] or [mg/cm^2]

ρ_a = the density of air ($1.29x10^{-3}$ g/cm^3)

R_t = the range of the alpha in tissue [cm] or [mg/cm^2]

ρ_t = the density of tissue ($1.0g/cm^3$)

Note that the units of range in media other than air are not pure distances, but rather are distances normalized to the density of the absorbing material, which is a surrogate quantity for the density of electrons through which the alpha particle will traverse as it creates ion pairs and becomes neutralized to become a helium atom. This normalization process is commonly done in radiation protection to make the results of any calculations less dependent on the specific characteristics of the absorbing medium. As we progress through the problems in this chapter and this book, this will be seen to be a common, and very useful, technique.

Through the use of only these equations, we can solve for a wide variety of variables, combinations of variables that have physical significance, or ratios of variables. The following example problems will illustrate some of these possibilities.

Problem 3-1

What is the energy of an alpha particle that has a range of 4.0 cm in air?

Step 1: Isolate the unknown variable:

$R_a = 1.24\ E - 2.62$ (based on the fact that ranges >3 cm in air indicate E>4MeV)

$$E = \frac{R_a + 2.62}{1.24}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$E = \frac{4.0 + 2.62}{1.24}$$

Step 5: Solve for the unknown:

E = 5.34 MeV

Step 6: Conduct reality check:

Confirm from alpha range/energy graph which indicates $E \approx 5.5$ MeV. There is terrible agreement if the equation for E<4MeV was used.

Problem 3-2

How much distance in air must be allowed to completely absorb a mixture of 2 MeV and 4 MeV alphas?

Step 1: Isolate the unknown variable:

$R_a = 0.56E$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful

Step 4: Plug in known quantities:

$R_a = 0.56 \bullet 4$

Step 5: Solve for unknown:

$R_a = 2.2$ cm

Step 6: Conduct reality check:

Use $R_a = 1.24\, E - 2.62 \Rightarrow 2.34$ cm, or use graph $\Rightarrow \approx 2.4$ cm.

Note: Only the range of the higher energy alpha is of importance in this calculation.

Problem 3-3

If the range of an alpha particle is 4 cm in air what would it be in water?

Step 1: Isolate the unknown variable:

$R_{H_2O} = R_a\, \rho_a$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{g}{cm^2} = \text{cm} \bullet \frac{g}{cm^3}$$

Step 4: Plug in known quantities:

$$R_{H_2O} = 4 \bullet 1.29 \text{ x } 10^{-3}$$

Step 5: Solve for the unknown:

$$R_{H_2O} = 5.16 \text{ x } 10^{-3} \text{ g/cm}^2 = 5.16 \text{ x } 10^{-3} \text{ cm}$$

Step 6: Conduct reality check:

The answer would be expected to come out in the range less than about 10^{-2} cm, since it is known that an alpha will not penetrate the outer layer of skin and its effective thickness is in this range.

Problem 3-4

What is the range in tissue of the most energetic alpha from Pu-239 (5.16 MeV)?

Step 1: Isolate the unknown variable:

$$R_a \rho_a = R_t \rho_t$$

$$R_t = \frac{R_a \rho_a}{\rho_t}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\text{cm} = \frac{cm \bullet g \bullet cm^3}{cm^3 \bullet g}$$

Step 4: Plug in known quantities:

$$R_t = \frac{3.7 \bullet 1.29x10^{-3}}{1.0}$$ (R_a from Exhibit 3-2)

Step 5: Solve for the unknown:

$$R_t = 4.77 \times 10^{-3} \text{ cm}$$

Step 6: Conduct reality check:

As with the last problem, the easiest benchmark to put on an alpha range is its ability to penetrate the outer layer of skin. Here the range is below that 10^{-2} cm value.

Beta Particles

In the case of beta attenuation, there are several things that are different than for alpha. First, when we look at the number of negative betas vs. absorber thickness, as shown in Exhibit 3-3, we observe a range, but without the extended plateau that characterized the same data for alphas.

The first reason for this difference is that betas are not emitted monoenergetically from the nucleus. The shape of the energy spectrum for betas is shown in Exhibit 3-4.

Note the difference between the maximum energy of betas [E_{max}] and the average energy of the betas [E_{av}], which is generally assumed to be $E_{max}/3$. The second reason is that betas have the same mass and charge as the electrons with which they are interacting, increasing the probability that truly elastic collisions can take place resulting in radical changes in direction and energy.

As in the case of the alphas, empirical mathematical relationships between the maximum energy of the betas and the maximum range have been established. Probably the most famous statement of this relationship is known as Feather's Rule. There are alternative statements that are used, but Feather's covers the largest beta energy range, and is probably the most often used. This equation is:

$$R\ [\text{g/cm}^2] = 0.542\ E_{max} - 0.133 \text{ for } E_{max} > 0.6 \text{ MeV} \qquad \text{Equation 3-5}$$

where,

R = the range of the beta in the medium

E_{max} = the maximum energy of the betas

It should be noted that the physics discipline that is normally applied to the assignment of units to quantities in equations does not necessarily apply when working with empirical equations. The unit assignment criterion used in empirical cases is generally, "what

Exhibit 3-3.
Beta Range

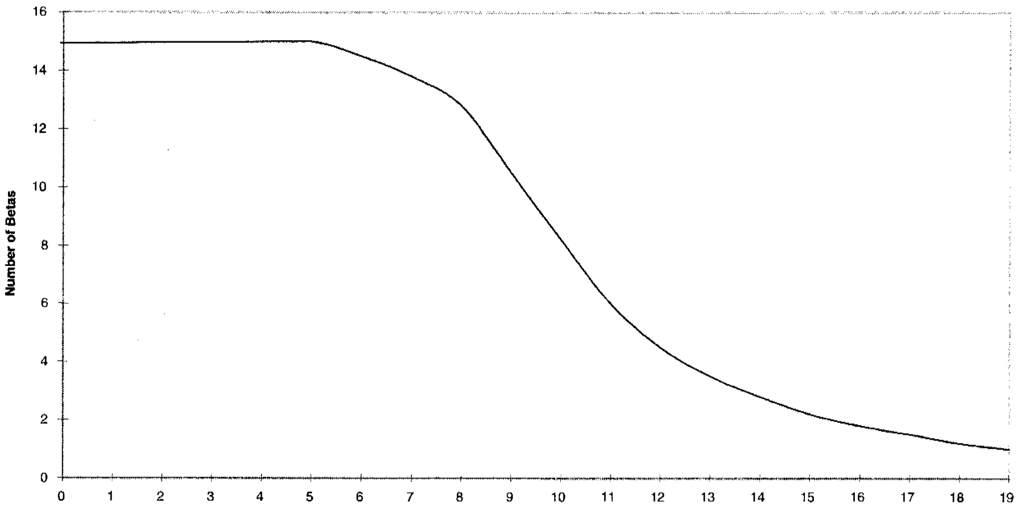
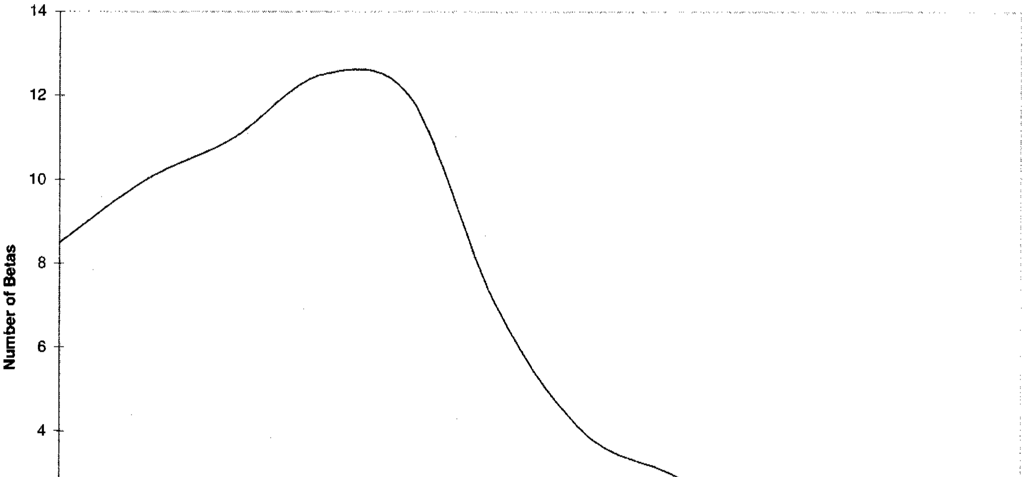


Exhibit 3-4.
Beta Energy Spectrum



quantities can I plot to make the results of experimental observations come out a straight line?". Therefore, whereas in the case of the alphas the unit of range is the intuitive [cm], for betas, the preferred unit of range is [g/cm^2]. It can be seen from Exhibit 3-5 that the range/energy plot for betas approximates a straight line.

As pointed out in the discussion of alpha ranges, the [g/cm^2] unit makes the quantity relatively independent of the material doing the absorbing because we have normalized to the density of the material. However, since the equation is empirical, and the units cannot be derived from first principles, it is imperative that the variables be addressed in the units presented here. Any deviation from this approach will certainly yield erroneous results. It should be noticed that there are no beta range equations with the range in terms of [cm], although we will calculate ranges in [cm] through unit conversion.

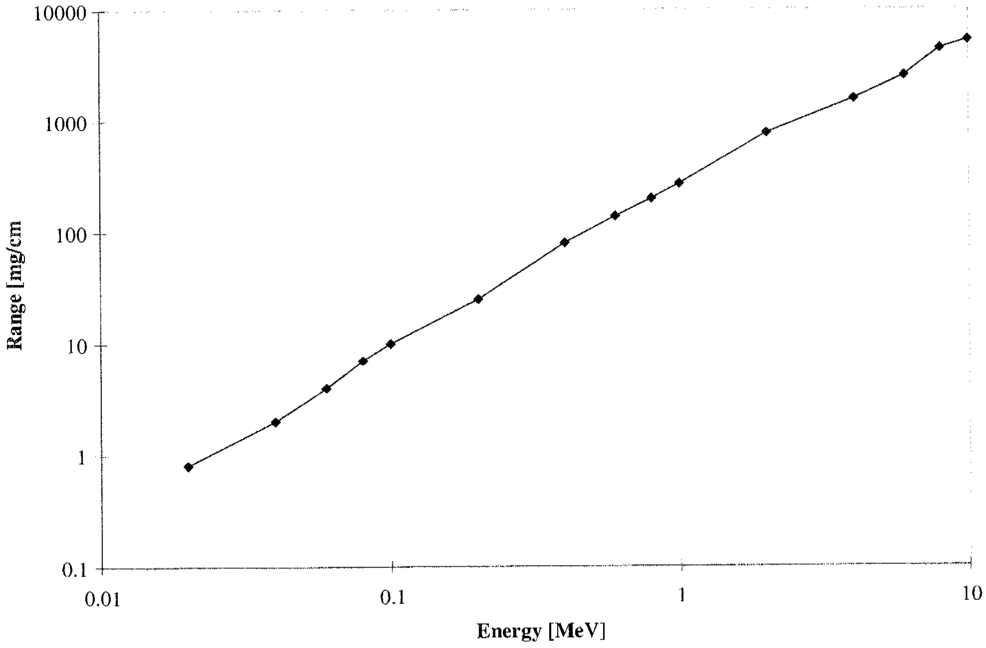
In addition to the mathematical and graphical approaches to determining beta ranges, there are some fairly commonly used rules of thumb available for use. The two most common rules of thumb are:

$R\ [g/cm^2] = E_{max}/2$ (in air) Equation 3-6a

R [ft] = 12 ft per MeV (in air) Equation 3-6b

There is another implication of absorbing high-energy betas in heavy materials which is the ability of betas to produce x-rays, as illustrated in Chapter 2 (sometimes called bremsstrahlung). This radiation results from the conservation of energy as the electron is

Exhibit 3-5.
Beta Range vs. Energy



decelerated by the absorbing medium. In contrast to the shielding of alphas by placing absorber of thickness greater than the range in that material, we must be cognizant of the fact that photon radiation may persist after a beta absorber thickness greater than its maximum range is employed. The importance of this issue can be evaluated by use of the following equation which gives the fraction of the total beta energy that is converted to photons:

$$F = 3.5\text{x}10^{-4}\ ZE \qquad \text{Equation 3-7}$$

where,

F = the fraction of the beta energy that goes into the production of photons

Z = the atomic number of the absorbing material

E = the maximum energy of the betas

If the material of interest is a compound or mixture the effective atomic number is calculated: by using the same approach as is presented in Equation 3-3, by replacing the A variable with Z.

If the proportion of energy converted to photon radiation is large, as it often is for high energy betas and heavy shielding materials, then shielding these photons must be considered separately. The techniques for solving these types of shielding problems will be covered in the "Exponentials and Logarithms" Part of this book.

The last aspect of betas that we need to consider is positive betas. These emissions come from nuclei that are proton rich, whereas negative betas come from fission products which are neutron rich. These positive betas are the antiparticle to negative betas. Therefore, when a positive beta annihilates a negative beta, the result is two photons of 0.51 MeV energy. Therefore, the shielding of these photons will be covered in the "Exponentials and Logarithms" Part.

Problem 3-5

What is the maximum range in Al in [cm] of a 4 MeV beta?

Step 1: Isolate the unknown variable:

$$R_m = 0.542\ E_{\max} - 0.133$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$R_m = 0.542 \bullet 4 - 0.133$$

Step 5: Solve for the unknown:

$$R_m = 0.542 \bullet 4 - 0.133 = 2.035 \text{ gm/cm}^2$$

Step 6: Confirm by graph and divide by density of Al (ρ_m = 2.7 gm/cm^3)

$$R_m = \frac{2.035}{\rho_m} = \frac{2.035 gm/cm^2}{2.7 gm/cm^3} = 7.54 \text{ x } 10^{-1} \text{ cm}$$

Problem 3-6

What is the density of a material that exhibits 1/3 the range for a 2 MeV beta, as does water?

Step 1: Isolate the unknown variable:

$$R_m = \frac{0.542E_{\max} - 0.133}{\rho_m}$$

$$R_{H_2O} = \frac{0.542E_{\max} - 0.133}{\rho_{H_2O}}$$

Step 2: Simplify the equation:

$$\frac{R_m}{R_{H_2O}} = \frac{\rho_{H_2O}}{\rho_m}$$

Step 3: Validate the problem setup:

The ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{1}{3} = \frac{1}{\rho_m}$$

Step 5: Solve for the unknown:

$$\rho_m = 3 \text{ g/cm}^3$$

Step 6: Conduct reality check:

Confirm using fact that $R_m \propto 1/\rho_m$

Problem 3-7

What is the energy of a beta that has a range of 5 x 10^3 mg/cm^2 in Al?

Step 1: Isolate the unknown variable:

$$R_m = 0.542\ E_{\max} - 0.133$$

$$E_{\max} = \frac{R_m + 0.133}{0.542}$$ (Al absorber is irrelevant)

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$E_{\max} = \frac{5.0 + 0.133}{0.542}$$

Step 5: Solve for the unknown:

$$E_{\max} = 9.47 \text{ MeV}$$

Step 6: Conduct reality check:

Confirm by graph or rule of thumb.

Problem 3-8

What proportion of a 5 MeV beta energy would go into photon production with a Pb absorber?

Step 1: Isolate the unknown variable:

$F = 3.5 \times 10^{-4} ZE$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$F = 3.5 \times 10^{-4} \bullet 82 \bullet 5$

Step 5: Solve for the unknown:

$F = 1.44 \times 10^{-1}$ (fraction is unitless)

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

CHAPTER 4

PHOTON BEHAVIOR

In addition to particles, ionizing radiation exists in the form of photons. Photons are packets of electromagnetic radiation. There are two distinct forms of ionizing radiation photons: gamma rays and x-rays. Gamma rays are photons that originate in the nucleus of an unstable atom. X-rays originate outside the nucleus of an atom. This chapter discusses photon behavior and how this behavior translates into radiation protection practices.

Wave Properties and Quantum Theory

Energy that is transmitted by disturbing a medium, allowing the disturbance to travel through the medium is represented by a wave. For example, when a stone is dropped into a pond, the kinetic energy of the stone is transmitted to the pond medium, the water. The disturbance moves through the water, causing the water to move up and down in a cyclic rhythm. This cyclic motion is the wave. The number of cycles that the wave goes through in one second is the frequency of the wave. The unit for frequency is the Hertz [Hz]. One Hz is one cycle per second. The distance that one cycle of the disturbance travels through the medium is the wavelength, measured in meters. The velocity of the disturbance, as it moves through the medium is the product of the wavelength (λ) and frequency (f).

$$v = \lambda \bullet f \qquad \text{Equation 4-1}$$

Electromagnetic waves, such as radio waves, microwaves, and x- or gamma-rays can be characterized by Equation 4-1. The velocity of electromagnetic waves equals the speed of light, 3×10^8 meters per sec (m/s) in a vacuum, which is represented by the variable c.

$$c = \lambda \bullet f \qquad \text{Equation 4-2}$$

Although wave mechanics are adequate to represent most electromagnetic behavior, it cannot represent all behavior. Quantum theory, or quantum mechanics, fills the gaps in wave theory in predicting the behavior of photons. Quantum theory states that electromagnetic energy is made up of packets of energy, called photons, which travel at the speed of light. The energy of a photon is given by Equation 4-3, where E is the energy of the photon and h is Planck's constant, 6.614×10^{-34} Joule-seconds [J-s].

$$E = h \bullet f \qquad \text{Equation 4-3}$$

By rearranging Equation 4-2, we find that the frequency, f, is the ratio of the speed of light to the wavelength. Making this substitution, Equation 4-3 becomes:

$$E = h \bullet \frac{c}{\lambda} \qquad \text{Equation 4-4}$$

Problem 4-1

What is the energy, in electron volts [eV], of an x-ray with the wavelength of 5×10^{-12} m? What is the frequency of this x-ray? 1 eV = 1.6×10^{-19} J, c = 3×10^{8} m/s, h = 6.614×10^{-34} J-s

Step 1: Isolate the unknown variable:

a) $E = h \bullet \frac{c}{\lambda}$

b) $c = \lambda \bullet f \Rightarrow f = c / \lambda$

Step 2: Simplify the equation:

The equations are simplified.

Step 3: Validate the problem setup:

E (eV) = J-s • m/s / m. The seconds (s) and meters (m) cancel leaving eV = J. This equation has units of energy on both sides, but not the same units. The Joules need to be converted to eV ⇒ eV = J • eV/J = eV✓

c (m/s) = λ (m/cycle) • f (cycles/s). The cycles cancel. m/s = m/s ✓

Step 4a: Plug in known quantities:

$$E = 6.614x10^{-34} J - s \bullet \frac{1eV}{1.6x10^{-19} J} \bullet \frac{3x10^{8} m/s}{5x10^{-12} m}$$

Step 5a: Solve for the unknown:

E = 248,000 eV = 248 keV

Step 4b: Plug in known quantities:

$f = 3 \times 10^{8}$ m/s / 5×10^{-12} m/cycle

Step 5b: Solve for the unknown:

$f = 6.0 \times 10^{19}$ cycles/s = 6.0×10^{19} Hz

Step 6: Conduct reality check:

In terms of eV, Plank's constant is approximately 4 x 10^{-15} eV-s. Equation 4-3 has the energy as the product of the frequency and Plank's constant. The product of 4 and 6 is 24 and the product of 10^{-15} and 10^{19} is 10^4. The energy should be about 24 x 10^4 eV or 240 keV. ✓

Gamma Radiation

Gamma rays are an example of electromagnetic radiation. Other examples of electromagnetic radiation include visible light, infrared radiation, radio waves, and microwaves. Gamma rays originate from the nucleus of an atom. They are highly penetrating, meaning that they are able to travel long distances through air and many other materials.

Some atoms are naturally unstable, but following a nuclear reaction, such as the absorption of a neutron, a stable nucleus of an atom can become unstable. This means that the reaction produces a nucleus that has excess energy. To become stable, the nucleus releases alpha or beta particles. Additional energy is lost by emitting a pulse of electromagnetic radiation, a gamma ray. The energy of the gamma ray and the frequency of its emissions are characteristics unique to a radionuclide.

Like all forms of electromagnetic radiation, the gamma ray has no mass and no charge. Gamma rays interact with material by colliding with the electrons in the shells of atoms. They lose their energy slowly as they travel through a material, enabling them to travel significant distances before stopping. Depending on their initial energy, gamma rays can travel from 1 to hundreds of meters in air and can easily go right through people. As a result, gamma rays require more shielding material, such as lead or steel, to reduce their intensity than is required for alpha and beta particles. Details on gamma ray interaction and shielding will be discussed in Chapter 11, *Interaction of Radiation and Matter.*

This chapter examines the behavior of a point source of gamma radiation and the associated gamma radiation field, or intensity of gamma radiation at a point away from the source. This intensity called exposure and is measured in roentgens [R]. The exposure rate is measured in roentgens per hour [R/hr]. Exposure is a measure of ionizations produced in air by x- or gamma-rays and is the sum of all electric charges of one sign produced in a mass of air. One roentgen is equal to 2.58 x 10^{-4} coulombs of charge per kilogram of air.

The gamma-ray intensity from a point source exhibits a "one-over-r-squared" relationship as you move away from the point source. In other words, the exposure rate decreases by a factor of d^2 as you move a distance d from the source. The equation for this behavior is presented below.

$$\frac{X_1}{X_2} = \frac{d_2^2}{d_1^2} \qquad \text{Equation 4-5}$$

where,

X_1 = exposure rate at a distance 1 (d_1) from a point source
X_2 = exposure rate at a distance 2 (d_2) from a point source

Problem 4-2

The exposure rate 2.5 meters from a point source of gamma radiation is 80 milliroentgen per hour (mR/hr). What is the exposure rate 5 meters from the source?

Step 1: Isolate the unknown variable:

$$\frac{X_1}{X_2} = \frac{d_2^2}{d_1^2} \quad \Rightarrow \quad X_2 = \frac{X_1 \bullet d_1^2}{d_2^2}$$

Step 2: Simplify the equation:

X_2 mR/hr = X1 mR/hr • (d_1 meters)^2 / (d_2 meters)^2

Step 3: Validate the problem setup:

mR/hr = mR/hr • square meters / square meters ⇒ the square meters cancel to get mR/hr = mR/hr ✓

Step 4: Plug in known quantities:

X_2 mR/hr = 80 mR/hr • (2.5 m)^2 / (5 m)^2 = 80 mR/hr • (2.5 m)^2 / (5 m)^2

Step 5: Solve for the unknown:

X_2 mR/hr = 80 mR/hr • 6.25 m^2 / 25 m^2 = 20 mR/hr

Step 6: Conduct reality check:

The exposure rate decreases as the square of the distance. In this problem, the distance is doubled, so the dose rate should decrease by a factor of 2 squared, or four. 20 mR/hr is one-fourth of 80 mR/hr. ✓

Problem 4-3

The exposure rate 1 meter from a point source of gamma radiation is 700 microroentgen per hour [μR/hr]. At what distance will the exposure rate be 10 μR/hr?

Step 1: Isolate the unknown variable:

$$\frac{X_1}{X_2} = \frac{d_2^2}{d_1^2} \quad \Rightarrow \quad d_2^2 = \frac{X_1 \bullet d_1^2}{X_2}$$

Step 2: Simplify the equation:

$(d_2 \text{ m})^2 = X_1$ μR/hr • $(d_1 \text{ meters})^2$ / X_2 μR/hr

Step 3: Validate the problem setup:

square meters = μR/hr • square meters / μR/hr ⇒ the exposure rates cancel to get square meters = square meters ✓

Step 4: Plug in known quantities:

$(d_2 \text{ m})^2$ = 700 μR/hr • $(1 \text{ meters})^2$ / 10 μR/hr

Step 5: Solve for the unknown:

$(d_2 \text{ m})^2$ = 700 μR/hr • $(1 \text{ meters})^2$ / 10 μR/hr ⇒ 70 m^2

$$d_2 = \sqrt{70m^2} = 8.4 \text{ meters}$$

Step 6: Conduct reality check:

The exposure rate decreases as the square of the distance. In this problem, the exposure rate decreases from 700 to 10 μR/hr, a factor of 70. The square of 8 is 64, which is close to 70. ✓

As a rule-of-thumb, the exposure rate from a point source of a single radionuclide one meter from the source is the product of 0.5 times the energy emitted from the radionuclide and the quantity of the radionuclide, measured in curies.

$$X = 0.5 \bullet C \bullet E \qquad \text{Equation 4-6}$$

where,

X = exposure rate 1 meter from the point source, in R/hr
0.5 = conversion factor, in R/hr per curie per MeV
C = quantity of the radionuclide, in curies
E = the energy of the emitted gamma radiation, in MeV

Problem 4-4

What is the exposure rate 1 meter from a 5 mCi of Cs-137? The energy emitted by Cs-137 is 0.662 MeV. 5 mCi of Co-60? Co-60 emits two gamma rays, one of 1.33 MeV and one of 1.17 MeV.

Step1 Isolate the unknown variable:

$X = 0.5 \bullet C \bullet E$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem setup:

$R/hr = R/hr/(Ci - MeV) \bullet Ci \bullet MeV \Rightarrow R/hr = R/hr$ ✓

Step 4: Plug in known quantities:

For Cs-137: $X\ R/hr = 0.5\ R/hr/(Ci - MeV) \bullet 5 \times 10^{-3}\ Ci \bullet 0.662\ MeV$
For Co-60: $X\ R/hr = 0.5\ R/hr/(Ci - MeV) \bullet 5 \times 10^{-3}\ Ci \bullet (1.33 + 1.17)\ MeV$

Step 5: Solve for the unknown:

For Cs-137: $X\ R/hr = 0.001655\ R/hr = 1.7\ mR/hr$
For Co-60: $X\ R/hr = 0.00625\ R/hr = 6.3\ mR/hr$

Step 6: Conduct reality check:

The exposure rate is one-half the product of the curie content and energy. The energy of the Cs-137 gamma is slightly more than one-half MeV, so the exposure rate should be a little more than one-half of one-half of the curie content of 5 mCi. One quarter (one-half of one-half) of 5 is 1.3. For Co-60, the total gamma energy is 2.5 MeV, which is about 4 times the Cs-137 energy. The Co-60 exposure rate is about four times the Cs-137 exposure rate. ✓

The rule-of-thumb provided by Equation 4-6 approximates the gamma constant or Γ factor for a gamma-emitting radionuclide. Exhibit 4-1 below presents the Γ Factors for some common gamma emitting radionuclides. These Γ factors give the exposure rate, in R/hr 1 meter from a point source. Equation 4-5 can be used to determine the exposure rate at any other distance.

Exhibit 4-1.
Gamma (Γ) Factors

Radionuclide	Γ Factor, R/hr @ 1 meter per Ci	Radionuclide	Γ Factor, R/hr @ 1 meter per Ci
Sb-125	0.27	I-131	0.22
Cs-134	0.87	Ir-192	0.48
Cs-137	0.33	Fe-59	0.64
Cr-51	0.16	Mo-99	0.18
Co-60	1.32	Ra-226	0.83
Eu-152	0.58	Ru-106	0.17
Eu-154	0.62	Ag-110m	1.43
Eu-155	0.03	Na-22	1.20
Au-198	0.23	Xe-133	0.01
I-125	0.07	Zr-95	0.41

Source: *Radiological Health Handbook*, U.S. Department of Health, Education, and Welfare, Jan. 1970.

X-rays

X-rays are an example of electromagnetic radiation that arises as electrons are deflected from their original paths or inner orbital electrons change their orbital levels around the atomic nucleus. X-rays, like gamma rays, travel long distances through air and many other materials. Like gamma rays, X-rays require more shielding to reduce their intensity than do beta or alpha particles. X- and gamma-rays differ primarily in their origin; x-rays originate in the electronic shell while gamma rays originate in the nucleus.

Characteristic x-rays are emitted when electrons drop from one electron orbital shell to a shell closer to the nucleus. Orbital electron capture and internal conversion are two radioactive decay mechanisms that include generating characteristic x-rays. In both of these decay mechanisms, electrons from inner orbital shells interact with the nucleus, resulting in an electron vacancy. As other electrons move into these vacancies, characteristic x-rays are emitted.

Another example of x-ray production is in an x-ray machine used for medical diagnostics or therapy. In these machines, electrons are accelerated through a potential difference and impinged on a metal target. This process produces some monoenergetic and some continuous x-rays. Although these x-rays have a benefit to the patient, other people at the x-ray facility must be protected from the x-rays.

In determining the amount of shielding required from the primary x-ray beam, a "K" factor is determined. K is given by the following:

$$K = \frac{d^2 P}{WUT}$$ Equation 4-7

where,

d = distance from the metal target on the x-ray machine to the area needing shielding from the x-rays

P = permissible weekly exposure, taken here to be 0.1 R/week (a fifty week exposure at this dose rate will equal to NRC external dose limit of 5 rem per year, assuming that 1 R = 1 rem)

W = workload of the machine, in milliamp-minutes

U = use fraction, the fraction of the time that the beam is pointed towards the area needing shielding while operating

T = occupancy factor, the fraction of the time that the area needing shielding is occupied. For full time occupancy, T = 1; for partial occupancy, T = 0.25; for occasional occupancy, T = 0.0625.

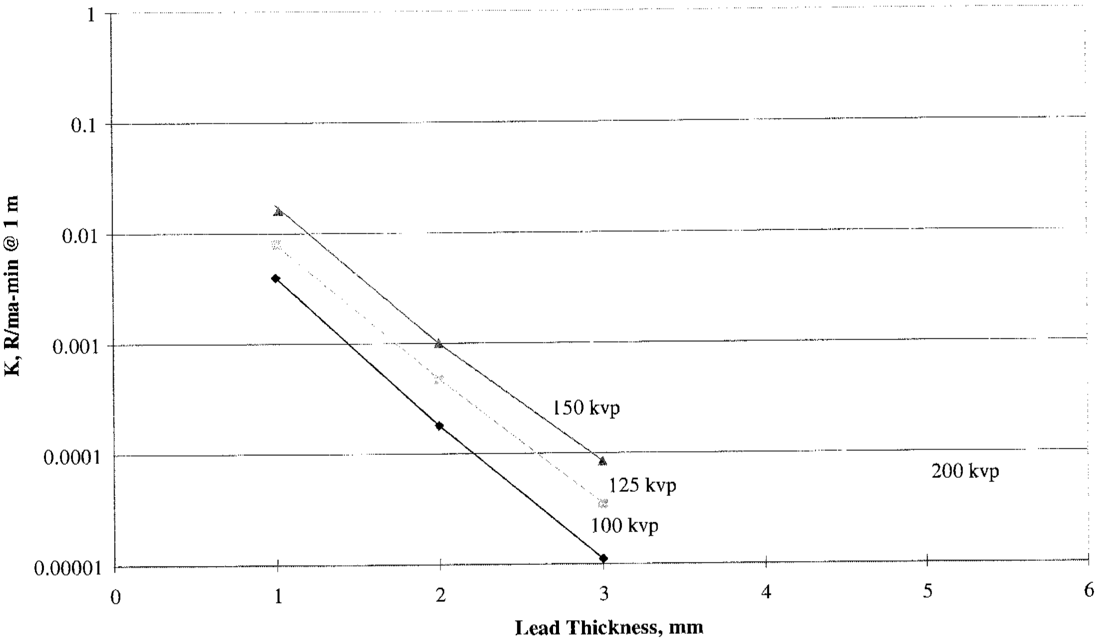
Areas considered to have full time occupancy include waiting rooms, offices, and control spaces. Partial occupancy areas include corridors, utility rooms, and parking lots. Areas with occasional occupancy include stairways, elevators, and closets.

The K factor is a convenient quantity for determining the required shielding for an area. Publications, such as the *Radiological Health Handbook* (Public Health Service Publication No. 2016, U.S. Department of Health, Education, and Welfare, 1970) and *Structural Design and Evaluation for Medical Use of X-Rays and Gamma Rays of Energies up to 10 MeV* (NCRP Report No. 49, 1976) present graphs of K values versus material thickness for common shielding material and X-ray energy.

Problem 4-5

The primary beam of an x-ray machine points towards a corridor. The wall between the corridor and the machine is 2.6 meters away. The beam is pointed towards this wall 100 percent of the time. The workload of the machine is 27,000 milliamp-minutes per week. If the beam potential is 100 kvp, how much lead shielding is required on the corridor wall? If the beam potential is 200 kvp? Use the graph (Exhibit 4-2) below to determine the shielding thickness from the calculated values of K.

Exhibit 4-2.
X-Ray Attenuation by Lead



Step 1: Isolate the unknown variable:

$$K = \frac{d^2 P}{WUT}$$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem setup:

R/ma-min @ 1 m = (m^2 • R/week) / (ma-min/week • unitless • unitless) ⇒ The "week" unit cancels out. Also, there is an implied 1 meter squared in the denominator of the equation, so the m^2 term also cancels (that is what gives is the "@ 1 m"). R/ma-min @ 1 m = R/ma-min @ 1 m ✓

Step 4: Plug in known quantities:

$$K = \frac{(2.6m)^2 \bullet 0.1R/week}{27{,}000ma - \min/week \bullet 1 \bullet 0.25}$$

Step 5: Solve for the unknown:

K = 0.0001 R/ma-min @ 1 m

From the graph above, for a 100 kvp beam, approximately 2.3 mm of lead is needed. For a 200 kvp beam, 4.5 mm of lead is needed.

Step 6: Conduct reality check:

The most common way to determine shielding requirements for x-ray facilities is from these graphs of K versus shielding thickness for different beam potentials. The graph included with this problem is an example of an actual graph, but is not an actual representation of true data. Resources such as the *Radiological Health Handbook* (Public Health Service Publication No. 2016, U.S. Department of Health, Education, and Welfare, 1970) and *Structural Design and Evaluation for Medical Use of X-Rays and Gamma Rays of Energies up to 10 MeV* (NCRP Report No. 49, 1976) present graphs of K values versus material thickness for common shielding material and X-ray energy.✓

A third example of x-ray production is Bremsstrahlung radiation. Bremsstrahlung are x-rays generated when charged particles undergo rapid deceleration. For example, when a beta particle travels near a nucleus of an atom, the electric attraction of the opposite charges causes the beta particle's path to bend. This phenomenon is important in radiation protection. Beta particles do not pose a radiation hazard when the beta emitting radionuclide is outside the body. However, when shielding beta radiation, Bremsstrahlung x-rays could be generated. Since x-rays are penetrating radiation, the resulting Bremsstrahlung could pose a radiation hazard.

The fraction of the beta particle's energy converted to Bremsstrahlung x-rays is given by:

$$F = 3.5 \times 10^{-4} \bullet Z \bullet E \qquad \text{Equation 4-8}$$

E is the maximum energy of the beta particle, in MeV, and Z is the atomic number of the material that the beta particle is traveling in. From Equation 4-8, it can be seen that the higher the atomic number, the greater the fraction of beta particle energy converted to x-ray energy (and the greater potential for posing a radiation safety hazard). Note that Equation 4-8 is empirical and provides an estimate for the Bremsstrahlung hazard. The constant, 3.5×10^{-4} has units of inverse MeV.

Problem 4-6

What fraction of the incident beta energy from S-35 is converted to Bremsstrahlung x-rays when shielded by aluminum (Z=13)? Shielded by lead (Z=82)? What material is the better shield for S-35, a pure beta emitter? The maximum beta energy for S-35 is 0.1674 MeV.

Step 1: Isolate the unknown variable:

$F = 3.5 \times 10^{-4} \bullet Z \bullet E$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem setup:

F (unitless) = (1/ MeV) • (unitless) • MeV ⇒ unitless = unitless ✓

Step 4: Plug in known quantities:

For aluminum: $F = 3.5 \times 10^{-4}$ 1/MeV • 13 • 0.1674 MeV
For lead: $F = 3.5 \times 10^{-4}$ 1/MeV • 82 • 0.1674 MeV

Step 5: Solve for the unknown:

For aluminum: $F = 7.6 \times 10^{-4}$
For lead: $F = 4.8 \times 10^{-3}$

Step 6: Conduct reality check:

The product of Z and E for aluminum is slightly more than 2, so F should be slightly more than twice the constant of 3.5×10^{-4}. ✓ The Z for lead is more than 6 times the Z for aluminum, so the F for lead should be more than 6 times that of aluminum. ✓

CHAPTER 5

ACCELERATORS

Accelerators impart large amounts of kinetic energy to charged particles that in turn bombard target materials. Usually, these charged particles are accelerated in a vacuum then brought out of the evacuated region to bombard the target. This bombardment generates other nuclear particles or exotic radionuclides. Often, stray or extraneous radiation (characteristic x-rays, prompt gamma radiation from the target, neutrons) is generated in the accelerator process. The accelerated particles, as well as the stray radiation, pose a radiation hazard to operators.

Additionally, accelerators are used to generate charged particle beams used for medical therapy. In these accelerators, the target is a portion of the body and the charged particles themselves provide the therapeutic benefit.

Electric Potential

As with x-ray machines discussed in Chapter 4, accelerators use electric or magnetic fields to impart kinetic energy on charged particles. Remembering from classic physics, particles of a particular charge (positive or negative) are attracted to an oppositely charged particle or plate. Similarly, charged particles subjected to a magnetic field will move. These principles govern the operation of accelerators.

From the study of electricity, the quantity of electric charge is measured in coulombs [C]. The charge of a proton or electron is the same in magnitude, 1.6×10^{-19} C and opposite in sign. The electron has a negative charge and the proton a positive charge. Neutrons, which exist in an atom's nucleus with protons, have no charge.

Current is the movement of charged particles. The amount of current is measured in amperes [amps]. One amp is equal to a coulomb per second [C/s]. 6.25×10^{18} electrons moving through a point in a second would generate a current of 1 amp.

Charged particles of different sign are attracted to one another while charged particles of like sign are repulsed. As a result, if you hold one charged particle stationary and place another charged particle near it, there is an electric force between them. Particles of the same sign will want to move farther apart while particles of the different sign will want to move together. To keep the particles at a constant distance from one another, work must be performed. Similarly, when the charged particles move together or apart, work is performed by the particles. The amount of work or energy exerted per amount of charge is called the potential difference existing between the particles and is measured in volts. One volt is equal to exerting one joule of energy on one coulomb of charge:

$$V = W/q \qquad \text{Equation 5-1}$$

where,

V = potential difference in volts
W = work or energy, in joules
q = charge, in coulombs

Problem 5-1

How much work is required to move one electron across a potential difference of one volt.

Step 1: Isolate the unknown variable:

$V = W/q$
Multiply both sides by q $\Rightarrow V \bullet q = W$

Step 2: Simplify the equation:

$W = V \bullet q$

Step 3: Validate the problem setup:

W (joules) = V (volt) • 1 (joule/coulomb per volt) • q (coulomb) ⇒ J = J ✓

Step 4: Plug in known quantities:

$W = 1\ \text{V} \bullet 1\ \text{J/C/V} \bullet 1.6 \times 10^{-19}\ \text{C}$

Step 5: Solve for the unknown:

$W = 1.6 \times 10^{-19}\ \text{J}$

Step 6: Conduct reality check:

One electron subjected to a potential difference of one volt is the definition of an electron volt [eV]. One electron volt is equal to 1.6×10^{-19} J. ✓

As illustrated in the problem above, the electron volt is a measure of energy. The eV is a common measure of energy in nuclear applications. However, the unit is very small. Often, the energy is in terms of million electron volts [MeV] or thousand electron volts [keV].

Accelerator Operation

As discussed earlier, accelerators subject charged particles to a potential difference. This electrical energy is then converted to kinetic energy of the moving particle. The beam of the accelerator, which is composed of the accelerator's charged particles, is measured in current, or amps. The current can be converted to the number of charged particles passing a point in the beam (for example, striking a target) in a second of time.

I [amps] = q [C] / time [s] Equation 5-2

# of particles/second = [q (C) /time (s)] • # of particles/coulomb Equation 5-3

There are 1.6×10^{-19} C of charge per electron or proton. There is also 1.6×10^{-19} C of charge per deuteron (a hydrogen ion with one proton and one neutron) or triton (a hydrogen ion with two neutrons and one proton ⇒ a tritium ion). A helium nucleus (an alpha particle) has a charge of 3.2×10^{-19} C, since it has two protons. For heavier ions, the quantity of charge, in coulombs, is the product of the number of protons and 1.6×10^{-19} C.

Problem 5-2

How many tritons strike a target per second given a 0.1 μamp cyclotron beam?

Step 1: Isolate the unknown variable:

I (amps) = q (C) / time (s)
# of particles/second = (q [C] /time [s]) • # of particles/coulomb

Step 2: Simplify the equation:

To solve this system of two equations, substitute I into the second equation for q/time.

# of particles/second = I [amps, or C/s] • # of particles/coulomb

Step 3: Validate the problem setup:

# of particles/second = I [amps, or C/s] • # of particles/coulomb ⇒
# of particles/second = # of particles/second ✓

Step 4: Plug in known quantities:

# of particles/second = 0.1×10^{-6} amps • 1 triton/1.6×10^{-19} coulomb

Step 5: Solve for the unknown:

# of particles/second = 6.3×10^{11} tritons/s

Step 6: Conduct reality check:

One amp is one C/s or 6.3×10^{18} tritons per second. 10^{-7} amps would be 6.3×10^{11} tritons per second. ✓

When particles strike a target, often radiation is emitted. This radiation is emitted isotropically, in other words in all directions. To determine the flux of the radiation, in particles or photons per unit area per second at a point, you must look at a sphere with its center at the target and radius equal to the distance from the target to the point of interest. The area of a sphere with radius r is:

$$4 \bullet \pi \bullet r^2 \qquad \text{Equation 5-4}$$

To determine the flux at a point, you divide the number of particles or photons emitted from the target per second (the production rate) by the surface area of a sphere through the point of interest.

Problem 5-3

If the beam of tritons in Problem 5-2, above, generates 0.01 neutrons per triton when it strikes the target, what is the neutron flux 2 meters from the target?

Step 1: Isolate the unknown variable:

$$flux = \frac{production\ rate\ (neutrons\,/\,s)}{4 \bullet \pi \bullet r^2}$$

# of neutron/second = # of tritons/s • 0.01 neutrons per triton

Step 2: Simplify the equation:

To solve this system of two equations, substitute the second equation in for the production rate.

$$flux = \frac{\#\ of\ tritons\,/\,s \bullet 0.01\ neutrons\ per\ triton\ (neutrons\,/\,s)}{4 \bullet \pi \bullet r^2}$$

Step 3: Validate the problem setup:

# of neutrons/second/m^2 = # of neutrons/second/m^2 ✓

Step 4: Plug in known quantities:

$\text{\# of neutrons/s/m}^2 = 6.3 \times 10^{11}$ tritons/s • 0.01 neutrons/triton/$\{4 \bullet \pi \bullet (2\text{m})^2\}$

Step 5: Solve for the unknown:

$\text{\# of neutrons/s/m}^2 = 1.3 \times 10^8$ neutrons/s/m^2

Step 6: Conduct reality check:

One neutron is emitted for every 100 tritons hitting the target. A sphere with a radius of 2 meters has a surface area of about 50 m^2 so the neutron flux should be about 1/5000th of the triton beam intensity, or about 1×10^8 neutrons/s/m^2. ✓

In the problem above, the answer was expressed in neutrons per second per square meter. Commonly, particle or photon flux is expressed in particles (or photons) per second per square centimeter. Since there is 100 centimeters in a meter, there is 10,000 square centimeters in a square meter (100 squared). The answer for Problem 5-3 would become 1.3×10^4 neutrons/s/cm^2 (the answer from above divided by 10,000 cm^2/m^2).

CHAPTER 6

MEASUREMENT OF RADIATION

For purposes of this discussion, there are two basic types of radiation detectors: capacitor-type and pulse-type. Typical capacitor-type detectors are the self-reading pocket dosimeter and the condenser R meter. Examples of the pulse-type are ionization, G-M, and proportional-based instruments, as well as scintillation and most other solid-state devices. Since both the operational fundamentals and the radiation applications of these two groups of instruments are quite different, they will be dealt with separately in this chapter.

Capacitance-Type Instruments

As the name implies, the operation of this type of instrument is based on the capacitor, which stores charge in an electric field, but which is discharged when exposed to ionizing radiation. Furthermore, the extent to which it is discharged is proportional to the amount of ionizing radiation to which it is exposed, independent of dose rate at normally encountered dose rates where these dosimeters are used. The relationship among the capacitance of a capacitor, or dosimeter [C], and the stored charge [Q], and the applied voltage [V] is :

$$C\ [\text{Farads}] = Q\ [\text{Coulombs}] / V\ [\text{volts}] \qquad \text{Equation 6-1}$$

The mechanism for discharging the capacitor during exposure to an ionizing radiation is the migration of the resulting ion pairs to the capacitor electrodes. The ion pairs all take the same amount of energy to create, to a first approximation, and all exhibit the charge of an electron. These two quantities are constants, as defined in the problem solving technique adopted in this book. Also, the mass of air is required for solving these problems because it is involved in the Roentgen definition of the quantity of ionizing radiation to which air (detector chamber) is exposed. Values to be used for these quantities in the following problems are as follows:

- charge on an electron = $1.6\text{x}10^{-19}$ [coulombs]
- energy to create an ion pair (or free an electron) = 35 [electron volts (eV)]
- definition of Roentgen [R] = 87.7 [ergs/gram] = $2.58\text{x}10^{-4}$ [coul/kg]
- $1.6\text{x}10^{-6}$ [ergs/MeV]
- 10^{6} [eV/MeV]
- density of air at STP [ρ_a] = $1.29\text{x}10^{-3}$ [grams/cm^3]

These variables, constants, conversion factors and definitions are used in the design, calibration, operation and interpretation of outputs from capacitor-type detectors used for radiation protection purposes. This will be demonstrated in the following example problems.

Problem 6-1

A pocket ionization chamber with a capacitance of 3 (ρF) and a volume of 1 cm^3 is initially charged to 150V. What exposure to gamma radiation will discharge the chamber to 130 V?

Step 1: Isolate the unknown variable:

$$C = \frac{Q}{V}$$

$$Q = CV$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

[coulomb] = [Farad][Volt]

Step 4: Plug in known quantities:

$$Q_1 = 3 \text{ x } 10^{-12} \bullet 1.5 \text{ x } 10^{2} = 4.5 \text{ x } 10^{-10} \text{ coul}$$

$$Q_2 = 3 \text{ x } 10^{-12} \bullet 1.3 \text{ x } 10^{2} = 3.9 \text{ x } 10^{-10} \text{ coul}$$

$$\Delta Q = 4.5 \text{ x } 10^{-10} - 3.9 \text{ x } 10^{-10} = 0.6 \text{ x } 10^{-10} \text{ coul}$$

$$0.6 \text{ x } 10^{-10} \text{ coul} \bullet \frac{IP}{1.6x10^{-19}\,coul} = 0.375 \text{ x } 10^{9} \text{ IP}$$

$$0.375 \text{ x } 10^{9} \text{ IP} \bullet \frac{35eV}{IP} \bullet \frac{MeV}{10^{6}eV} \bullet 1.6 \text{ x } 10^{-6}\,\frac{erg}{MeV} \bullet \frac{Rg}{87.7erg} \bullet \frac{cm^{3}}{1.29x10^{-3}g} \bullet \frac{1}{cm^{3}}$$

Step 5: Solve for the unknown:

$$1.8 \text{ x } 10^{-1} \text{ R} = 180 \text{ mR}$$

Step 6: Conduct a reality check:

A 180 mR exposure is a realistic outcome, based on experience with such chambers.

Problem 6-2

A standard thimble chamber is exposed to a radiation field and loses a charge of 3 x 10^{-11} coul in 10 seconds. If the volume of the chamber is 1.2 cm^3, what is the exposure rate in [R/sec]?

Step 1: Isolate the unknown variable:

Unit analysis.

Step 2: Simplify the equation:

Unit analysis.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{kgR}{2.58x10^{-4}coul} \bullet \frac{3x10^{-11}coul}{10\sec} \bullet \frac{1}{1.2cm^3} \bullet \frac{cm^3}{1.29x10^{-3}g} \bullet \frac{10^3 g}{kg}$$

Step 5: Solve for the unknown:

$$7.5 \text{ x } 10^{-3} \frac{R}{\sec}$$

Step 6: Conduct reality check:

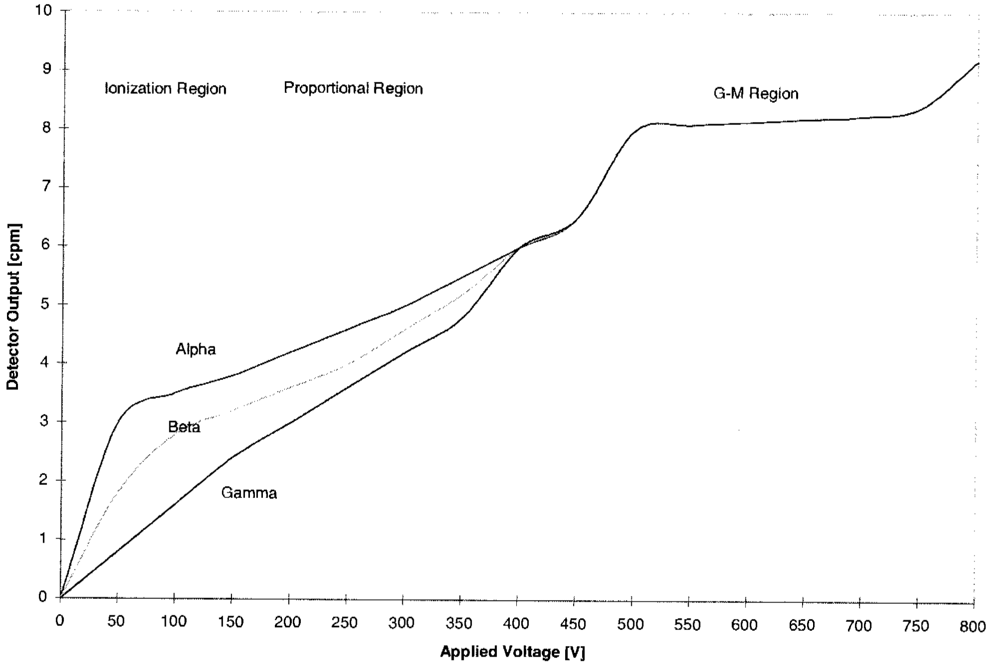
An outcome in the range of mR/sec is realistic based on experience with such chambers.

Pulse-Type Instruments

As their name implies, pulse-type instruments create pulses of current when exposed to ionizing radiation. These pulses can be counted individually, can be integrated over time, or integrated over other characteristics (like energy) to yield important radiation protection information.

For the gas filled tubes of this type, it is easily observable, across the complete range of operational characteristics, that the more voltage that is applied across the electrodes, the greater the current; and the more intense ionizing radiation field they are exposed to, the greater the current. If one plots output pulse height vs. Voltage applied to the detector, starting at zero volts, the curve in Exhibit 6-1 results.

Exhibit 6-1.
Ionization-Type Detector Response



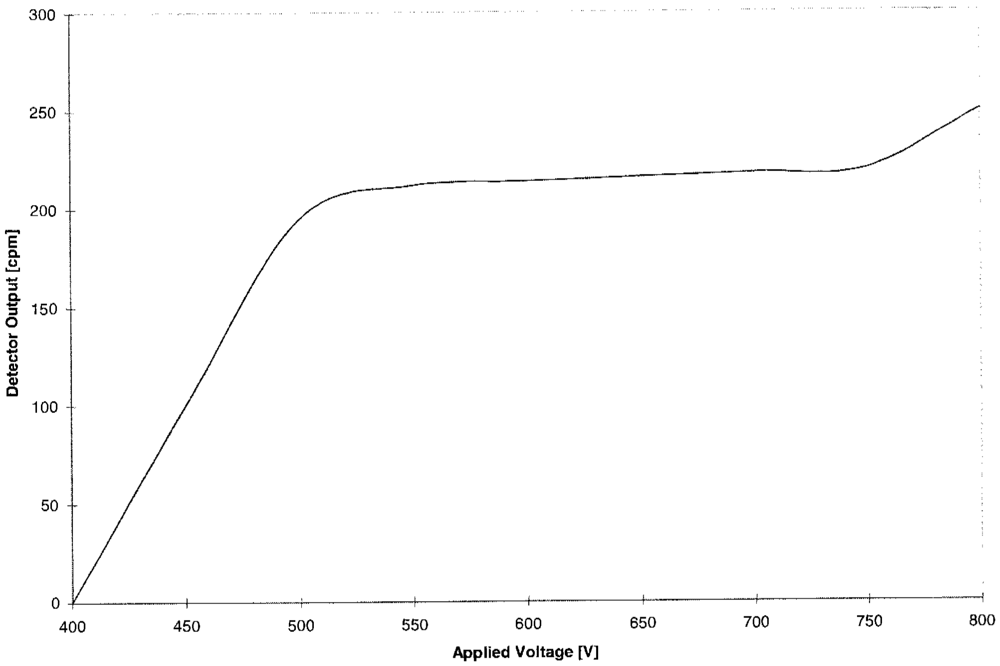
The first abrupt rise in pulse height with increased voltage is the result of increasingly efficient collection of the ions being created by exposure to the ionizing radiation field. It is assumed that the intensity of the radiation field is constant throughout the collection of these data. The first plateau is called the ionization region and here the ions created are being totally collected on the electrodes. It should be noticed that this condition produces the collection of more alpha ions than beta ions because alpha particles create many more ions per unit track length in the chamber.

The next higher voltage region on the Exhibit is called the proportional region. Here, the ion pairs created directly by the incident ionization radiations are multiplied by the electric field in the chamber, but the output current is proportional to the amount of energy originally deposited in the chamber.

The highest voltage plateau on the Exhibit is the Geiger-Mueller region. Here, the initial ions are greatly multiplied by the applied electric field, and because of the magnitude of this multiplication, the output becomes independent of the amount of energy initially deposited in the chamber. At voltages higher than the G-M region, the ability of gas in the tube to insulate one electrode from the other is lost, and a continuous discharge (arcing) situation is established and the tube is rendered useless.

Looking only at the G-M region of the previous curve, and plot counts per minute [cpm] vs. Voltage, the curve shown in Exhibit 6-2 results.

Exhibit 6-2.
G-M Detector Response



There are two primary aspects of interest in this curve. The first is in terms of answering the question, "At what voltage should the detector be operated?" It should be noticed that in the lower voltage ranges no counts per minute result. The answer is usually something like "1/3 to ½ of the way between V_1 and V_2" which defines the extent of the plateau region. This is where variations in count rates with normally small voltage changes are minimal.

The second interest is related to the first in that it will determine how true the last statement is for any specific detector. This question is "What is the slope of the curve on the plateau?" Because this question is so important and, in fact, is a prime criterion for accepting or rejecting a detector for use, specifications have been set in defining this slope. This specification is that the slope be assessed in the units of [%/100volts]. Therefore, using the variables noted on the Exhibit, the definition of the slope is:

$$\text{Slope} = \{(C_2 - C_1)/100\} \bullet 10^4/(V_2 - V_1) \qquad \text{Equation 6-2}$$

Notice that this equation conforms to the classical form of slope = rise/run. For most detectors of this type a slope in the range 3% or below is judged to be good. If the slope of the plateau is much greater than this acceptable value, then the variations of count rate with small variations in voltage are large enough to be unacceptable for most radiation protection uses. Once the appropriate operating voltage is determined for a detector, then the interest turns to the Calibration Factor or Efficiency of the detector. This quantity is defined by:

CF [%] = {NCPM [net counts/minute]/DPM [disintegrations/minute]} • 100 Equation 6-3

where,

NCPM = CPM (sample) - CPM (background)

This is different than the "intrinsic efficiency" which is the probability that a count will be recorded if radiation enters the sensitive volume. This CF considers the geometry factors involved in the radiation getting to the detector as well as the probability that the radiation will be able to penetrate to the sensitive volume.

The point was made earlier in this book that it is important to be able to solve any of the equations discussed for any of their variables, given sufficient information. This is particularly true for this equation, because occasions for using the equation to determine all of the variables are nominally equal. As will be demonstrated by problems to follow, instrument calibrations to standard sources involve using the equation as stated. Establishing the unknown activity of a source involves using the known efficiency and the observed ncpm.

This comparison to a standard approach is also used to determine the mass of a radionuclide that is present in a sample. The quantity involved in these determinations is called "specific activity", which is defined as the activity [Ci] per gram [g] of material. The technique is based on the fact that (1) the activity of a specific nuclide is proportional to the number of atoms present and (2) that a mole of any element contains Avogadro's Number of atoms (6.02×10^{23} atoms). In calculating these quantities we use the following equations:

$A = -\lambda N$ [disintegrations/unit time] Equation 6-4

where,

A = the disintegrations per unit time of the radionuclide

$\lambda = 0.693/\text{half life}$ [1/unit time] Equation 6-5

where,

N = number of atoms of radionuclide present in the sample

$$g/gmw = N/N_A \qquad \text{Equation 6-6}$$

where,

g = grams of radionuclide in the sample

gmw = gram molecular weight (atomic mass number)

N = number of atoms of radionuclide present is the sample

N_A = Avogadro's Number

Whether the radiation protection problem calls for converting activity to mass or mass to activity, the first step is to calculate N, the number of radionuclide atoms in the sample. If the activity of the sample is given this calculation is done using Equation 6-4, making sure that the time unit in the activity [dps or dpm] quantity is the same as the time unit for the half-life. Following the determination of N, in this typical problem, the mass is calculated using Equation 6-6, knowing N from the initial calculation, the gmw of the radionuclide and the constant N_A.

If the problem is reversed and the mass is known and the activity is the unknown, N is calculated from Equation 6-6, knowing the mass in grams, gmw and N_A. Once N is known the activity of the sample is calculated using Equation 6-4, knowing N and the half-life, which yields λ when divided by 0.693. Again, it is important to note that the time units of λ and A must be the same. To convert Ci, or some fraction of a Ci, usually the time unit is the second for the half-life and 1/second is the unit for λ.

Since this type of detector comes in many shapes, sizes and designs, it is highly important to understand that the current that results from an exposure to an ionizing radiation field will depend on the operating pressure and temperature of the detector. This comes about because the number of ions produced per unit path length of the ionizing radiation is dependent upon the number of gas molecules it encounters per unit path length. The number of molecules it encounters is determined by the density of the gas, the pressure at which the gas is in the detector and the operating temperature of the gas. For a detailed understanding of this concept, additional study of Boyle's Law is recommended.

For purposes here, it will be sufficient to recollect what is probably intuitive and common knowledge about the behavior of gases. First, only a certain amount of gas can be put into a container at a specified pressure and temperature. If more gas is to be put into that volume, more pressure must be applied. It is also true that a reduction in the temperature of the gas will allow more gas to be put into the specified volume. Conversely, if, given the same volume, one wishes to reduce the gas pressure, less gas will be able to occupy the container. Likewise, an increase in temperature will either result in an increase in pressure or gas will have to be vented from the container to keep the pressure constant. The mathematical statement of this principle is

(Pressure • Volume)/Temperature = constant   Equation 6-7

To evaluate the relevance of this concept to currents from radiation detectors resulting from exposure to ionizing radiation, and other applications, a Standard Temperature and Pressure [STP] has been defined. These quantities will be useful in working radiation detector problems. The definitions are:

Standard Temperature = 273 degrees Kelvin [^{o}K]; ^{o}K = ^{o}C + 273

Standard Pressure = 360 mm Hg or 1 atmosphere

The degree to which deviations from STP affect the current flowing from a detector as a result of exposure to a radiation is evaluated by forming the ratio of standard pressure to the actual pressure and the actual temperature to standard temperature, remembering that an increase in pressure will result in an increase in current and an increase in temperature will result in a decrease in current, because of the effect of these two conditions on the number of gas molecules a radiation will encounter per unit path length while in the detector. Mathematically this concept can be stated:

actual current = current at STP • (actual pressure/standard pressure) •
(standard temperature/actual temperature)   Equation 6-8

When working problems, one must be very careful to make sure that the proper pressure and temperature quantities are in the numerator and denominator of the calculation setup.

The response of gas filled detectors is also a function of the energy of the incoming radiations. At low energies radiations may not be able to penetrate the detector tube and/or shield. At intermediate energies phenomena beyond the scope of this book result in the detector either under- or overresponding. At higher energies normally detectors indicate the true radiation intensity. A quantity called the "Correction Factor" is used to communicate these over and underresponses and to enable the detector user to make the appropriate corrections. A typical Correction Factor curve is shown in Exhibit 6-3.
The Correction Factor is defined by the equation:

CF = True Reading / Meter Reading   Equation 6-9

where,

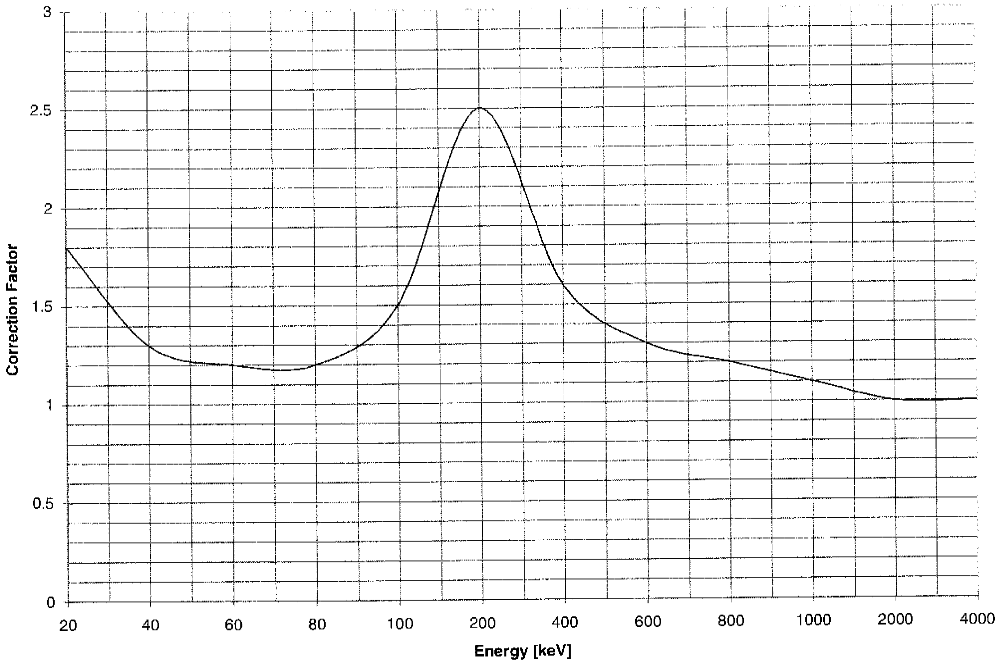
CF = Correction Factor [unitless]

True Reading = the true radiation intensity [in units of the meter reading]

Meter Reading = what the output of the detector instrument indicates [in units of mR/hr, etc.]

The Correction Factor curves for specific detector types are different but are supplied by the manufacturer to the user.

Exhibit 6-3.
Energy Dependence of Correction Factor



Problem 6-3

A radioactive sample is counted for 1 minute, yielding 100 counts [GC]. A background count is made for 10 minutes, yielding 100 counts [BC]. What is the net count rate [NCR]?

Step 1: Isolate the unknown variable:

NCR + BCR = GCR

NCR = GCR - BCR

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

[cpm] = [cpm] - [cpm]

Step 4: Plug in known quantities:

$$NCR = \frac{100}{1} - \frac{100}{10}$$

Step 5: Solve for the unknown:

$NCR = 90$ cpm

Step 6: Conduct reality check:

Check arithmetic.

Problem 6-4

Would you accept or reject measurements from a detector that exhibited a plateau characterized by $C_1 = 500$ cpm, $V_1 = 1000$ V, $C_2 = 600$ cpm and $V_2 = 1800$ V?

Step 1: Isolate the unknown variable:

$$\text{slope} = [\frac{(C_2 - C_1)/100}{V_2 - V_1}]10^4$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{cpm}{V} = \frac{cpm}{V}$$

Step 4: Plug in known quantities:

$$\text{slope} = [\frac{(600-500)/100}{1800-1000}]10^4$$

Step 5: Solve for the unknown:

slope = 12.5% $\Rightarrow$ reject measurements

Step 6: Conduct reality check:

Check arithmetic.

Problem 6-5

What is the efficiency of a detector that yields a count rate of 1 cps from a 100 pCi source?

Step 1: Isolate the unknown variable:

$$\text{CF} = \frac{cpm}{dpm} \text{ or } \frac{cps}{dps}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\text{dps} = 10^{2} \bullet 10^{-12} \text{ Ci} \bullet \frac{3.7x10^{10}\,dis}{Ci \sec} = 3.7 \text{ dps}$$

Step 5: Solve for unknown:

$$\text{CF} = \frac{1}{3.7} = 27\%$$

Step 6: Conduct reality check:

Efficiencies in this range are realistic.

Problem 6-6

What current will flow in a 10 cm ^{3} chamber when exposed to a 100 mR/hr radiation field at Standard Temperature and Pressure (STP)?

Step 1: Isolate the unknown variable:

Unit analysis.

Step 2: Simplify the equation:

Unit analysis.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{amp\sec}{coul} \bullet \frac{10^{-1}R}{hr} \bullet \frac{2.58x10^{-4}coul}{kgR} \bullet \frac{hr}{3600\sec} \bullet 10\ \mathrm{cm}^3 \bullet \frac{1.29x10^{-3}g}{cm^3} \bullet \frac{kg}{10^3 g}$$

Step 5: Solve for the unknown:

Current = 9.25×10^{-14} amp

Step 6: Conduct reality check:

Currents in the range of $\mu\mu$amps are realistic for these situations.

Problem 6-7

At what pressure is a 1 cm^3 chamber operating if exposure to a 40 R/hr field yields a current flow of 10^{-11} amp?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$10^{-11}\ \mathrm{amp} = 40\frac{R}{hr} \bullet \frac{2.58x10^{-4}coul}{kgR} \bullet 1\ \mathrm{cm}^3 \bullet \frac{1.29x10^{-3}g}{cm^3} \bullet \frac{kg}{10^3 g} \bullet \frac{hr}{3600\sec} \bullet \mathrm{P}$$

$$10^{-11}\ \mathrm{amp} = 3.7 \times 10^{-12} \bullet \mathrm{P}$$

$$\mathrm{P} = \frac{10^{-11}amp}{3.7 \times 10^{-12}amp}$$

Step 5: Solve for the unknown:

P = 2.7 atm

Step 6: Conduct reality check:

Chambers often operate at pressures in this range.

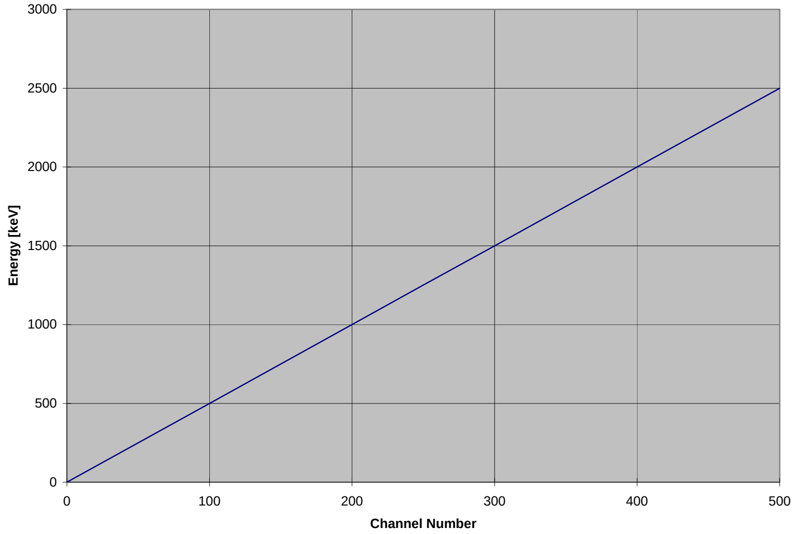
Solid State Detectors

One concept that is unique to solid state detectors is their ability to indicate the energies of incident radiations. However, for this type of information to be interpreted for beneficial purposes, the detector and its associated instrumentation must be calibrated, that is, the location of the output peaks from the detector on the energy scale must be calibrated to the known energy of the radiations that gave rise to the peaks. This calibration is in addition to the calibration of NCPM/DPM discussed in the last section.

In most cases, it can be assumed and confirmed that the relationship between channel number (or spectrometer setting) and energy is linear. This means that, theoretically, only two points need be specified before the line covering the whole energy spectrum can be drawn. In reality, several points are usually experimentally determined during the calibration process, to more accurately establish the location of the calibration line. Such a line is shown in Exhibit 6-4.

Once the location of the calibration line is established for a detector system, then the locations of unknown lines can be translated to energies with the objective of either identifying unknown radionuclides or determining the amount of radionuclide present in the sample being counted. The utility of this process is demonstrated in the following problems.

Exhibit 6-4
Energy Calibration Curve



Problem 6-8

If the calibration line for a NaI(Tl) detector is characterized as 10 keV/channel, what is the energy of a peak observed at channel 66?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$66 \text{ channel} \bullet \frac{10keV}{channel} \bullet \frac{MeV}{10^3 keV}$$

Step 5: Solve for the unknown:

Energy = 0.66 MeV

Step 6: Conduct reality check:

Energy recognizable as that of Cs-137.

Problem 6-9

If the 1.33 MeV peak from Co-60 is found in channel 1000, in what channel would its 1.17 MeV peak be expected?

Step 1: Isolate the unknown variable:

$$\frac{MeV_1}{channel_1} = \frac{MeV_2}{channel_2}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{1.33MeV}{1000channel} = \frac{1.17MeV}{channel_2}$$

Step 5: Solve for unknown:

channel $_2$ = 880

Step 6: Conduct reality check:

Check arithmetic.

Problem 6-10

What is the efficiency of a scintillation detector if exposure of the detector to a 10 pCi sample yields 1 cpm for a peak known to have a photon yield per disintegration of 80%?

Step 1: Isolate the unknown variable:

$$\text{CF} = \frac{cpm}{dpm}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\text{CF} = \frac{(\frac{1cpm}{0.8})}{dpm} \bullet 100$$

$$\text{dpm} = 10\text{pCi} \bullet 3.7 \text{ x } 10^{-2} \frac{dis}{pCi\,\text{sec}} \bullet \frac{60\,\text{sec}}{\text{min}} = 22.2 \text{ dpm}$$

Step 5: Solve for the unknown:

$$\text{CF} = \frac{\left(\frac{1}{0.8}\right)}{22.2} \bullet 100 = 5.63\%$$

Step 6: Conduct reality check:

Answer is in a range of reasonable efficiencies.

CHAPTER 7

ENVIRONMENTAL TRANSPORT AND SAMPLING

An important component of many radiation protection programs is environmental surveillance. It is the intent of this chapter to present a highly simplified analysis (ignoring radiological decay during transport) of how radionuclides are transported through surface water, groundwater and terrestrial systems. Atmospheric transport will be covered in the Exponential/Logarithm Part of this book. The objective here is to present problems that illustrate how environmental modeling and environmental sampling come together in the radiation protection process.

The first section of this chapter contains discussions of features of environmental transport that are independent of the transport medium involved. The second section discusses those features that are unique to each of the transport systems.

Transport Medium Independent Features

The most commonly used approach to environmental transport is called "pathway analysis". In this approach all of the receptors of radionuclides from the initial release to air or water, through all intermediate receptors like foodstuffs, to the final receptor-humans, are characterized by compartments. A diagram of one much studied pathway is shown in Exhibit 7-1.

Exhibit 7-1.
Environmental Pathway Diagram

I-131 Release
⇓
River
⇓
Irrigation
⇓
Grass
⇓
Cows
⇓
Milk
⇓
Man

In this pathway, there exists a hypothetical I-131 inventory which is released to a river, either in a chronic (slow release over a long time) or acute (larger release over a short time) release mode and transported by the river water to an irrigation system which supplies the contaminated water to grass, that is eaten by cows, that produce milk, that is consumed by man, resulting in a thyroid dose in humans.

Emergency responses are commonly based upon the dose consequence estimates that result from the release of a specified quantity of radionuclide. Environmental surveillance programs are commonly based on an understanding of the relative concentrations of radionuclides in various compartments of an environmental transport system, where radionuclides are likely to be found in high concentrations in the environment, and with which compartments humans interact directly. All of these endeavors require an understanding of the relationship of radionuclide concentrations in all compartments of the transport system with all other compartments, especially the magnitude of the release and the dose to man.

To illustrate the sequence used to calculate the concentrations in the compartments of an environmental transport system, we will examine the pathway from release of a radionuclide to a river, fish uptake of the radionuclide, and consumption of the contaminated fish by humans. The calculation sequence for this type of problem is:

- evaluate sourceterm
- calculate concentration in initial receptor
- calculate concentration in fish using concentration factor approach
- determine how much fish is consumed by maximum exposed individual (MEI)
- calculate dose to MEI

The evaluation of a sourceterm usually begins with a determination of the radionuclide inventory at risk. This could be Cs-137 inventory in an irradiated fuel element that fails, for instance. For catastrophic accidents, usually default values for a release fraction (defined as the ratio of the amount released to the amount in the inventory at risk) have been established by experimentation and/or modeling. In less severe, acute-type releases, often the effluent monitoring system will indicate the amount of radionuclide released. Almost always, in chronic release cases, either periodic sampling or effluent measurements will indicate the magnitude of releases.

In the fisherman scenario, since it takes a long time for the fish to come to concentration equilibrium with the radionuclides in the water, we will consider the release to be a chronic condition. Our sourceterm would be evaluated either by prerelease sampling or continuous monitoring. Let us assume that our monitoring indicates that the concentration of Cs-137 in our water effluent is $2\text{x}10^{-4}$ μCi/cm^3 and the flow rate is 10 liters/minute. Converting liters to cm^3 and multiplying these two values yields a release rate of 20 μCi/min. If the river is running at 10^9 cm^3/min, the concentration in the river is $2\text{x}10^{-8}$ μCi/cm^3 (20 μCi/min • min/10^9 cm^3) at the point where complete mixing of the release in the river flow has occurred.

It is known that fish can concentrate Cs-137 in their edible tissue to a point where the concentration in the tissue is 10^3 times that in the surrounding water. This factor is called the "Concentration Factor" and is defined by

$$\text{CF} = \text{(concentration in the medium of interest)/(concentration in previous compartment or host medium)} \qquad \text{Equation 7-1}$$

By multiplying the concentration in the water by the 10^3 concentration factor, we arrive at a concentration in the fish of 2×10^{-5} μCi/cm^3 or 2×10^{-5} μCi/gram of tissue. As will be covered in detail in the "Internal Dose" chapter of the Exponential/Logarithms Part of this book, the dose to a human resulting from an intake of radionuclide is calculated by multiplying the concentration of the radionuclide in the ingested material (fish, in this case) by the mass of the material ingested per unit of time, then by a dose factor. Assuming that our MEI eats 0.5 kg of these fish per day, his/her intake would be 10^{-2} μCi/day. Taking this intake rate times a dose factor [unit of mrem/μCi] one arrives at a dose per day estimate for the MEI. If at some point in time the ingestion exposure ceases, multiplying the dose per day by the number of days the exposure occurred, yields the dose to the MEI.

In some cases, all of the required information in not known for certain because of the lack of monitoring data, etc., so worst case default values for many transport quantities have been experimentally determined and are adopted as surrogate data when there are no known values for key quantities. For example, in the fisherman scenario, if the release rate for the Cs-137 were unknowable, one might assume that the Cs-137 was in the effluent stream at the Cs solubility limit in units of grams per liter. Using the specific activity calculation approach demonstrated in the *Measurement of Radiation* chapter, this gram-based quantity could be converted to a μCi/l value to replace the 2×10^{-4} μCi/cm^3 value we used, based on effluent measurement data.

Similarly, if data are insufficient to accurately calculate or determine the magnitude of releases from resuspension of contaminated soil or fire entrained contamination, many times a worst-case maximum airborne particulate loading of 10 mg/m^3 is used. Again, using the specific activity calculation approach, this can be converted to an airborne concentration in units of μCi/m^3 and propagated through the airborne transport system using the concentration factor approach demonstrated in the fisherman scenario for water.

For many of the more commonly analyzed release scenarios, default release fractions have been developed. These are radionuclide and release scenario specific and are tabulated in numerous accident analysis references.

Transport Medium Dependent Features

Surface Water --

As demonstrated in the fisherman scenario, concentrations in rivers are calculated assuming that the release concentration is diluted in the river volume to yield a lower concentration in the river water. It was also demonstrated in that scenario that there are concentration mechanisms that take the radionuclide in the river water and concentrate it to much higher values than exist in the water. Fish were given as an example in the fisherman scenario. Certain radionuclides are also highly concentrated in river sediments and in aquatic plants that may be ingested by fish that are then consumed by humans, or may be directly ingested by humans. If comprehensive and accurate, these transport pathway diagrams become very complex and are often referred to as "food webs".

Groundwater --

Since groundwater flows ever so much more slowly (can be as slow as cm/year) than surface water, flowing through small crevices, and therefore having time to interact with the ion exchange properties of the soil or geologic formation through which it is flowing, the determination of concentration of radionuclide in groundwater at a specific point is space and time is slightly different than for surface water. First, because of the ion exchange action of geologic media on most radionuclides, most do not travel "downstream" as fast as the groundwater flows. The ratio of the velocity of the groundwater flow to the velocity of radionuclide movement is called "K_d" and ranges from 1.0 to many thousands. Considering K_d, but ignoring the ion exchange process as a radionuclide REMOVAL mechanism, the radionuclide is diluted in less volume than would be estimated using the surface water technique. For example, if a specific radionuclide exhibits a K_d of 100, then the worst-case concentration would be calculated by assuming dilution in only 1/100 of the groundwater flow (assuming that the concentration limit for the radionuclide is not exceeded). Furthermore, if the distance from the release point to the point of interest is great, given insufficient time for the radionuclide to travel, this distance will preclude any radionuclide being ingested at that point until the radionuclide arrives. For example, if the groundwater of interest flows 1 km/year and the radionuclide has a K_d of 100, if the point of interest were 1 km from the release point, no radionuclide would be expected to arrive for 100 years. At that time, the worst case average concentration would be calculated by dividing the quantity of radionuclide released to the groundwater over the 100 years by the affected volume of groundwater between the release point and the point of interest. This concentration reflects a worst case because it ignores the radionuclide removed from the groundwater by both ion exchange and radioactive decay.

Terrestrial --

Terrestrial transport pathways are often more complex than water pathways because of the many more mechanisms that can (1) introduce contamination to the system, (2) the many compartments that may participate in radionuclide transport and (3) the many ways in which these compartments might interact, depending on the climatic characteristics of the location and the radionuclide of interest.

For instance, contamination might be introduced to a terrestrial pathway via a release from a stack to the atmosphere followed by deposition of the suspended particulates onto the ground or plant surfaces. In this case the concentration in a foodstuff is calculated by

$$C_f = C_{air} V_d F_t \qquad \text{Equation 7-2}$$

where,

C_f = concentration in the foodstuff [μCi/kg]

C_{air} = concentration in air [μCi/m^3]

V_d = deposition velocity [m/day] (can be assumed to be 1000 for most applications)

F_t = transfer factor defined as concentration in the foodstuff per unit deposition rate [μCi/kg or μCi/l per μCi/m^2-d] (ranges from 10^{-3} to 1.0)

Concentrations in animal products are calculated by (ignoring radioactive decay)

$$C_{ap} = F_{animal} (C_{forage} Q_f + C_{water} Q_w) \qquad \text{Equation 7-3}$$

where,

C_{ap} = concentration of radionuclide in animal product [μCi/kg or μCi/l]

F_{animal} = transfer factor of radionuclide defines as the fraction of daily intake transferred to the animal product [d/kg or d/l] (ranges from 10^{-5} to 10^{-1})

C_{forage} = concentration of radionuclide in forage [μCi/kg]

Q_f = quantity of forage consumed [kg/d]

C_{water} = concentration of radionuclide in water [μCi/l]

Q_w = quantity of water consumed [l/d]

A second frequent mechanism for introducing contamination to the terrestrial ecosystem is resuspension. In this case the concentration in the air is calculated through the use of a quantity called "resuspension factor" which is defined as

$$\text{Resuspension Factor} = \text{concentration in air } [\mu\text{Ci/m}^3] / \text{concentration ON soil } [\mu\text{Ci/m}^2] \qquad \text{Equation 7-4}$$

Resuspension factors are determined empirically and depend on climatic conditions, types of resuspension forces (wind, vehicular traffic), soil type and radionuclide.

Radionuclides may be introduced into plants directly from the soil. In this case, concentration factors, as defined in Equation 7-1, are used. These values are also widely tabulated. Ignoring radioactive decay, the radionuclide concentrations in soil resulting from, for instance, irrigation can be calculated by assuming buildup of the inventory in the surface layer (say top 10 cm) as more radioactivity is added by watering. The relevant data would likely take the form of a concentration in the irrigation water, the flow rate of the irrigation water was delivered to the soil and the area of the soil that was irrigated.

Calculation of airborne concentrations over space and time requires the use of exponentials and will be discussed in that part of this book.

Once these types of calculations are done and understood for released radionuclides, earlier model predictions can be validated, intakes by humans can be calculated and environmental surveillance sampling programs can be designed to (1) take maximum advantage of key pathway components that concentrate a released radionuclide to yield maximum sensitivity to releases and optimum statistical significance of resulting data, and (2) have the most direct interface with the ultimate human receptor, thus minimizing uncertainty in the final intake and dose components of the calculation.

Problem 7-1

If a radionuclide is released at a concentration of 10 μCi/cm ^{3} at a rate of 1 liter/sec to a sewer pipe with a flow rate of 10^4 liters/sec, at what concentration does the radionuclide reach the treatment plant? If this radionuclide is concentrated 10^5 times in the treatment plant sludge, what is its concentration in the sludge?

Step 1: Isolate the unknown variable:

Unit analysis.

Step 2: Simplify the equation:

Unit analysis.

Step 3: Validate the problem setup:

$$concentration\ \ factor \bullet \left(\frac{\mu Ci}{l\ of\ water} \bullet \frac{l}{1000g} \right) = \frac{\mu Ci}{g\ of\ sludge}$$

Step 4: Plug in known quantities:

$$\frac{10\mu Ci}{cm^3} \bullet \frac{10^3 cm^3}{\sec} \bullet \frac{\sec}{10^4 l} = 1.0 \frac{\mu Ci}{l} \text{ in water}$$

Step 5: Solve for the unknown (assuming that the sludge has the same density as water):

$$\frac{1\mu Ci}{l} \bullet 10^5 = 1 \text{ x } 10^5 \frac{\mu Ci}{l} \bullet \frac{l}{1000g} = 10^2 \frac{\mu Ci}{g} \text{ in sludge}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 7-2

If a process vessel holding 10^2 Ci of radionuclide fails and in an hour releases 10% of its inventory to a river flowing at 10^6 liter/minute, assuming complete mixing, what is the radionuclide concentration in the river? If an aquatic plant exhibits a concentration factor of 10^4, what is the radionuclide concentration in this plant?

Step 1: Isolate the unknown variable:

Unit analysis.

Step 2: Simplify the equation:

Unit analysis.

Step 3: Validate the problem setup:

$$(\textit{concentration factor} \bullet \frac{Ci}{g} \text{ in river}) = \frac{Ci}{g} \text{ in plant}$$

Step 4: Plug in known quantities:

$$10^2 \text{ Ci} \bullet 10^{-1} \bullet \frac{\min}{10^6 l} \bullet \frac{1}{60\min} = 1.67 \text{ x } 10^{-7} \frac{Ci}{l} = 1.67 \text{ x } 10^{-10} \frac{Ci}{g} \text{ in river}$$

Step 5: Solve for the unknown:

$$1.67 \text{ x } 10^{-10} \frac{Ci}{g} \bullet 10^4 = 1.67 \text{ x } 10^{-6} \frac{Ci}{g} \text{ in aquatic plant}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 7-3

If a radionuclide that exhibits a K_d of 10^3 is released to groundwater with a flow velocity of 2 m/year, how long would it take it to arrive a a groundwater well 1 km away?

Step 1: isolate the unknown variable:

$$K_d = \frac{velocity\ of\ groundwater}{velocity\ of\ radionuclide}$$

$$\text{velocity of radionuclide} = \frac{velocity\ of\ groundwater}{K_d}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\text{velocity of radionuclide} = \frac{\frac{2m}{y}}{10^3}$$

Step 5: Solve for unknown:

$$\text{velocity of radionuclide} = 2 \times 10^{-3}\ \frac{m}{y}$$

$$\frac{10^3 m}{\frac{2x10^{-3} m}{y}} = 5 \times 10^5 \text{ years for radionuclide to reach the well}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 7-4

If a parcel of soil is contaminated to a level of 10^{3} μCi/m^{2} and exhibits a resuspension factor of 10^{-2}, what is the concentration in alfalfa in the downwind direction if the deposition velocity is 10^{3} and the transfer factor is 10^{-2}?

Step 1: Isolate the unknown variable:

$$\text{RF} = \frac{\textit{concentration in air}}{\textit{concentration on ground}}$$

concentration in air = $RF \bullet$ concentration on ground

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{\mu Ci}{m^3} = \frac{\mu Ci / m^3}{\mu Ci / m^2} \bullet \mu\text{Ci/m}^{2}$$

Step 4: Plug in known quantities:

$$\text{concentration in air} = 10^{-2} \bullet 10^{3}$$
$$= 10\,\frac{\mu Ci}{m^3}$$

Step 5: Solve for the unknown:

$$\text{C}_F = \text{C}_A \text{V}_d \text{F}_t$$

$$= \frac{10\mu Ci}{m^3} \bullet 10^{3} \bullet 10^{-2} = 10^{2}\,\frac{\mu Ci}{g} \text{ in alfalfa}$$

Step 6: Conduct a reality check:

Make independent order of magnitude estimate.

Problem 7-5

What is the concentration in a cow's meat if it eats 10^{2} kg of hay contaminated at 1µCi/kg and drinks uncontaminated water if the transfer factor is 10^{-2}?

Step 1: Isolate the unknown variable:

$$C_{ap} = F_{a}(C_{f}Q_{f})$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{\mu Ci}{kg} = \frac{d}{kg}\left(\frac{\mu Ci}{kg} \bullet \frac{kg}{d}\right)$$

Step 4: Plug in known quantities:

$$C_{ap} = 10^{-2}(1 \bullet 10^{2})$$

Step 5: Solve for unknown:

$$C_{ap} = 1\frac{\mu Ci}{kg}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 7-6

What is the concentration in a garden vegetable if the contamination level of the soil is 10 µCi/kg and the concentration factor for the vegetable is 0.5?

Step 1: Isolate the unknown variable:

$$CF = \frac{\textit{concentration in vegetable}}{\textit{concentration in soil}}$$

concentration in vegetable = CF • concentration in soil

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{\mu Ci}{kg} = \text{unitless ratio} \bullet \frac{\mu Ci}{kg}$$

Step 4: Plug in known quantities:

concentration in vegetable = 0.5 • 10

Step 5: Solve for unknown:

$$\text{concentration in vegetable} = 5\ \frac{\mu Ci}{kg}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 7-7

What would the expected soil volumetric concentration be from irrigation of 100 m^2 of land with water contaminated at 1μCi/liter, if the irrigation rate is 10 liters/min, the wetting depth is 10 cm and irrigation lasts for 4 hours?

Step 1: Isolate the unknown variable:

Unit analysis.

Step 2: Simplify the equation:

Unit analysis.

Step 3: Validate the problem setup:

$$\text{concentration in soil} = \frac{activity \bullet wetting\ \ rate \bullet time}{area \bullet depth}$$

Step 4: Plug in known quantities:

$$\text{concentration in soil} = \frac{\frac{1\mu Ci}{l} \bullet \frac{10l}{\min} \bullet 240\min}{100m^2 \bullet 0.1m}$$

Step 5: Solve for unknown:

$$\text{concentration in soil} = \frac{2.4 \bullet 10^3\,\mu Ci}{10\,m^3} = \frac{2.4 \bullet 10^2\,\mu Ci}{m^3}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

CHAPTER 8

WASTE MANAGEMENT

Waste management encompasses a wide variety of activities that are relevant to radiation protection. A typical production cycle is shown in Exhibit 8-1. It includes producing a product from input materials with recycle and waste management, but is robust enough to illustrate decommissioning, etc., with very minor modification. Normally a facility's waste management plan is based upon the premise that no long-term buildup of waste inventory will occur onsite. Therefore, provision is made to direct release waste; store, then release; or treat, then ship waste; all with appropriate monitoring and/or sampling and analysis.

For radiation protection purposes, these possible waste management scenarios bring into play the following areas that may require calculations:

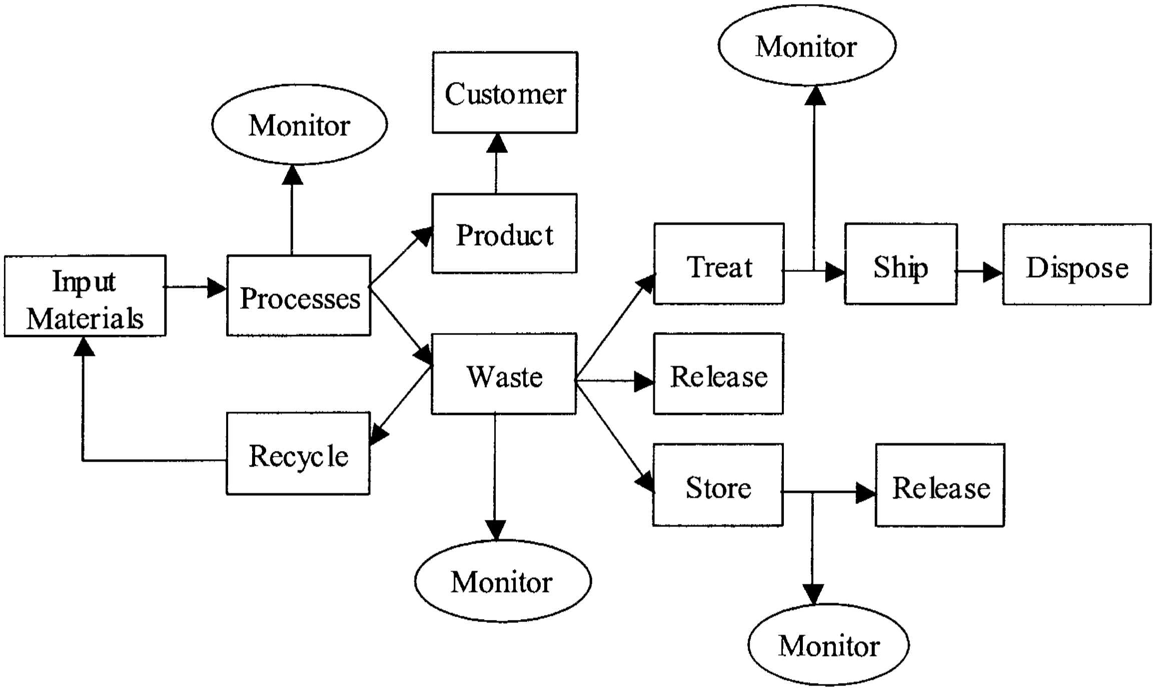
- classification of waste
- regulatory waste release criteria
- effluent stream cleanup
- transportation regulations for shipping
- waste acceptance criteria for waste disposal
- license limitations on stored inventory
- estimation of stored inventory vs. time
- monitoring/sampling waste solids, liquids and gases in effluents

Waste management includes such a broad range of possible combinations of activities that we will not attempt to illustrate a comprehensive treatment of the subject, but rather will highlight those topics that are most relevant to the knowledge base required of a certified radiation protection technologist.

Classification of Waste

Much of what can and cannot be done with waste is dictated by its classification. Radiological wastes may be classified as high or low-level, transuranic or mixed waste. Short definitions of these are given below. Legal definitions can be found in 40 CFR. Non-radiological hazardous waste is referred to simply as "hazardous waste".

Exhibit 8-1.
Process Flow Design



High-level Waste-contains fission products, traces of uranium and plutonium, and other transuranic elements, which result from the first extraction cycle of chemical reprocessing of spent fuel or spent fuel itself.

Low-level Waste-is all radioactive waste not classified as high-level waste, mixed or transuranic waste, spent fuel or byproduct material.

Mixed Waste-is low-level waste that contains both radioactive and hazardous components.

Transuranic Waste (TRU)-is waste contaminated with alpha-emitting radionuclides with half-lives of greater than 20 years, such as plutonium and americium, in concentrations greater than 100 nCi per gram of waste.

Historically the high, intermediate and low-level definitions were based on the dilution that would have to be accomplished for the waste to be released to the environment. Low-level wastes were those that required a dilution of no more than 10^3, intermediate 10^3 < dilution factor < 10^5, and high level > 10^5. These values are no longer applicable, but they give some technical context to the present legalistic definitions. Mixed and transuranic waste are newer concepts and have less tested historical bases. The classification of a waste or waste stream will determine which other regulations are applicable to its management. For example, low-level solid waste disposal is governed by 10 CFR61, whereas high-level waste disposal is governed by 10 CFR60, or applicable government agency regulations.

Problem 8-1

What class of waste would describe a 55 gallon drum full of soil contaminated with 5 Ci of Pu-239?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$55 \text{ gal} \bullet \frac{m^3}{2.64x10^2 gal} \bullet \frac{10^6 cm^3}{m^3} \bullet \frac{2g}{cm^3} = 4.17 \text{ x } 10^5 \text{ g}$$

Step 5: Solve for the unknown:

$$\frac{5x10^9 nCi}{4.17x10^5 g} = 1.2 \text{ x } 10^4 \frac{nCi}{g} > 100 \frac{nCi}{g} \therefore \text{Transuranic (TRU) Waste}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Regulatory Release Criteria and Waste Stream Cleanup

Many extremely low level waste streams can be released directly to the environment. The permissible concentrations are waste matrix and radionuclide specific and are promulgated in 10 CFR20. If a waste stream does not conform to the requirements for direct release, then treatment or storage for decay may be applied before release, or shipping offsite for disposal may be required.

It is often necessary to remove particulate matter from airborne effluent streams before release to the atmosphere. This is most often accomplished through filtration. Since the filtration efficiencies that are required for many radiation protection applications are so high (>99.99%) it has been found to be desirable to define a more "user friendly" filtration effectiveness quantity than filter efficiency. The quantity most often used as a measure of filter merit is called "decontamination factor", (df). This quantity is defined by the equation:

$$df = 100 / (100 - E) \qquad \text{Equation 8-1}$$

where,

E = filter efficiency [%]

For example, a filter with an efficiency of 99.9% would pass one particle per thousand and would have a df of 10^3, in contrast to a filter with a similar looking efficiency of 99.99% which has a df of 10^4.

If the contamination in a waste stream is radioactive and short lived, perhaps storage and decay is an acceptable waste management strategy. Radiological decay during waste storage is radionuclide specific and is described by a quantity called "half-life", that was briefly mentioned in the *Measurement of Radiation* chapter in the "specific activity" discussion. Half-life is defined as the time it takes a radionuclide to decay to ½ its original activity. An algebraic way to state this definition is:

$$A = A_0 / 2^n \qquad \text{Equation 8-2}$$

where,

n = the number of half-lives that separate A and A_0 in time.

A_0 = the original activity of the source

A = the activity of the source after the passage of n half-lives

Problem 8-2

If the concentration limit for releasing a radionuclide to a sewer system is 2×10^{-5} µCi/cm^3, could a waste stream flowing at 100 liters/min containing this radionuclide at a concentration of 1 µCi/liter be released at a rate of 0.1 liter/min?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{1\mu Ci}{l} \bullet \frac{10^{-1} l}{\min} = \frac{10^{-1}\mu Ci}{\min}$$

$$\frac{10^{-1}\mu Ci}{\min} \bullet \frac{\min}{100 l} = \frac{10^{-3}\mu Ci}{l}$$

$$\frac{10^{-3}\mu Ci}{l} \bullet \frac{l}{10^{3} cm^{3}} = \frac{10^{-6}\mu Ci}{cm^{3}}$$

Step 5: Solve for the unknown:

$$\frac{10^{-6}\mu Ci}{cm^{3}} < \frac{2x10^{-5}\mu Ci}{cm^{3}} \therefore \text{waste can be released}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 8-3

If a volume of radioactive gas being held up for decay in pressurized tanks has four times the release limit concentration at atmospheric pressure, how many half-lives will it need to be held before it can be released to the atmosphere?

Step 1: Isolate the unknown variable:

$$\frac{1}{2^{n}} = \frac{1}{4}$$

Step 2: Simplify the equation:

$$n = \sqrt{4}$$

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$n = \sqrt{4}$

Step 5: Solve for the unknown:

n = 2 half-lives

Step 6: Conduct reality check:

Check arithmetic.

Problem 8-4

What is the decontamination factor of a filter of 99.995% efficiency?

Step 1: Isolate the unknown variable:

$$\mathrm{df} = \frac{100}{100 - E}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\mathrm{df} = \frac{100}{100 - 99.995}$$

Step 5: Solve for the unknown:

$\mathrm{df} = 2 \times 10^4$

Step 6: Conduct reality check:

Check arithmetic.

Problem 8-5

Into what volume of uncontaminated liquid effluent would 100 liters of waste contaminated to 1 μCi/l have to be diluted to meet a 10^{-8} μCi/cm^3 release criterion?

Step 1: Isolate the unknown variable:

$$\frac{1\mu Ci}{l} \bullet 100\ l = 100\ \mu\text{Ci}$$

$$10^2\ \mu\text{Ci} \bullet \frac{1}{V[l]} \bullet \frac{l}{10^3 cm^3} = \frac{10^{-8} \mu Ci}{cm^3}$$

Step 2: Simplify the equation:

$$V[l] = 10^2 \bullet \frac{1}{10^3} \bullet \frac{1}{10^{-8}}$$

Step 3: Validate the problem setup:

$$l = \mu\text{Ci} \bullet \frac{l}{cm^3} \bullet \frac{cm^3}{\mu Ci}$$

Step 4: Plug in known quantities:

See Step 2.

Step 5: Solve for the unknown:

$V = 10^7$ liters

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 8-6

What is the concentration of an airborne waste stream contaminated to 1 μCi/cm^3 after it passes through a filter of df = 10^4 ?

Step 1: Isolate the unknown variable:

$$\text{df} = \frac{100}{100 - E}$$

$$E = -\frac{10^2}{10^4} + 10^2$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$E = 99.99\% \Rightarrow \text{Transmission} = 10^{-2}\%$ or 10^{-4}

$$\frac{1 \mu Ci}{cm^3} \bullet 10^{-4} = \frac{10^{-4} \mu Ci}{cm^3}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 8-7

What increase in the μCi/cm^3 for a solid waste occurs when it is compacted from 80 gallon to 0.5 gallon in volume? What is the change in μCi/g?

Step 1: Isolate the unknown variable:

$$\text{Concentration}_1 = \frac{activity_1}{volume_1}$$

$$\text{Concentration}_2 = \frac{activity_1}{volume_2}$$ (note that the activity does not change with compaction)

Step 2: Simplify the equation:

$$\text{Concentration}_2 = \text{Concentration}_1 \bullet \frac{volume_1}{volume_2}$$

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4-5: Plug in known quantities:

$$\frac{80}{0.5} = 160 \therefore 160 \text{ times increase in } \mu\text{Ci/cm}^3$$

No change in μCi/g because there is no change in the mass.

Step 6: Conduct reality check:

Check arithmetic.

Transportation Regulations

Transportation regulations are to be found in 49 CFR. These regulations govern what type of container must be used to transport wastes with specific characteristics, how much waste can be transported in each container, how the container must be marked, and the radiological conditions exterior to the container, including establishing the appropriate transport index.

Problem 8-8

What would a portable detector with a correction factor of 2 read at the surface of a vehicle transporting radioactive material as an exclusive use vehicle if the radiation field there was at the permissible limit?

Step 1: Isolate the unknown variable:

Limit = 200 mR/hr

$$\text{CF} = \frac{actual}{reading}$$

$$\text{reading} = \frac{actual}{CF}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\text{reading} = \frac{200}{2}$$

Step 5: Solve for the unknown:

reading = 100 mR/hr

Step 6: Conduct reality check:

Check arithmetic.

Problem 8-9

Calculate the transport index of an unshielded source of 1 g of Cs-137, half-life of 30 years, if 1 Ci of Cs-137 gives rise to a radiation field of 0.33 R/hr/Ci @1meter?

Step 1: Isolate the unknown variable:

Transport index is equivalent to the maximum radiation level in mR/hr @ 1 meter.

$$\frac{g}{gmw} = \frac{N}{N_A}$$

$$\frac{1}{137} = \frac{N}{6.02x10^{23}} \Rightarrow \text{N} = 4.39 \text{ x } 10^{21} \text{ atoms}$$

$$\text{A} = \lambda\text{N}$$

$$\text{A} = \frac{0.693}{(30y \bullet 3.15x10^7 \text{ sec}) / y} \bullet 4.39 \text{ x } 10^{21} = 3.22 \text{ x } 10^{12} \text{/sec}$$

$$3.22 \text{ x } 10^{12} \text{/sec} \bullet \frac{Ci}{3.7x10^{10}} \text{ sec} = 87 \text{ Ci}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Will be done while solving problem.

Step 4-5: Plug in known quantities:

$$87 \text{ Ci} \bullet \frac{0.33R}{hrCi} \approx 29 \frac{R}{hr} \Rightarrow 29{,}000 = \text{Transport Index}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Waste Acceptance Criteria

The waste acceptance criteria (WAC) establish the characteristics the waste received at a disposal site must possess, or not possess, to be permanently emplaced. Typical WAC categories for low level radioactive waste are given below.

Waste form

- combustability
- gas generation
- immobilization
- explosives and compressed gases
- pyrophorics
- toxic and corrosive materials
- sludges and free liquids

Container

- structure
- weight
- dimensions
- surface dose rate
- thermal power
- labeling
- surface contamination

WAC also exists for mixed, transuranic and high level radioactive wastes. These generally address the same basic points but are very specific to the waste type and available modes of permanent disposal.

Problem 8-10

Can 3 m^3 of soil contaminated with 100 Ci of Th-232 be sent to a disposal site that has a 1 mCi/kg limit on waste it can dispose?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities (assuming a soil density of 2 g/cm^3):

$$\frac{10^5 mCi}{2m^3} \bullet \frac{m^3}{10^6 cm^3} \bullet \frac{cm^3}{2g} \bullet \frac{10^3 g}{kg} = \frac{50mCi}{kg}$$

Step 5: Solve for the unknown:

$$\frac{50mCi}{kg} > \frac{1mCi}{kg} \therefore \text{ waste cannot be sent for disposal}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 8-11

If a distributed source of a radionuclide with half-life of 1 year and mass number of 100 gives an exposure rate of 20 mR/hr/mCi, what is the maximum mass of nuclide that can be shipped for disposal if the limit is 100 mR/hr?

Step 1: Isolate the unknown variable:

$$\frac{100mR}{hr} \bullet \frac{mCihr}{20mR} = 5 \text{ mCi}$$

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$A = \lambda N$

$$5 \text{ mCi} \bullet \frac{3.7x10^7 dis}{mCi \sec} = \frac{0.693}{1y} \bullet \frac{1y}{3.15x10^7 \sec} N$$

$N = 8.41 \text{ x } 10^{15}$ atoms

$$\frac{g}{gmw} = \frac{N}{N_A}$$

$$\frac{g}{100} = \frac{8.41x10^{15}}{6.02x10^{23}}$$

Step 5: Solve for the unknown:

$g = 1.4 \text{ x } 10^{-6}\text{ g}$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Limitations on Stored Inventory and Inventory vs. Time

As discussed previously in this chapter, in many cases, short-lived radioactive wastes do not conform to the requirements for immediate release, but can conform after a short onsite holdup/storage period of decay. For longer-lived radionuclide wastes that are generated fairly slowly in a facility, they may be stored onsite until a full load can be accumulated for shipment to a disposal site. In both cases, inventories may be limited by external license requirements or facility Operational Safety Requirements. With the possibility of continuous, simultaneous, additions and removals from the stockpile, frequent inventory evaluation is necessary to ensure compliance.

Problem 8-12

If waste is accumulated in a facility at the rate of 100 μCi/day, how many days can waste be stored if the license limit on inventory is 100 mCi, ignoring radiological decay?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$100 \text{ mCi} \bullet \frac{1d}{100\mu Ci} \bullet \frac{10^3 \mu Ci}{mCi} = \text{time}$$

Step 5: Solve for the unknown:

Time = 10^3 days

Step 6: Conduct a reality check:

Make independent order of magnitude estimate.

Monitoring

Characterization of wastes and verifying operational conditions require monitoring and/or sampling. These activities can take many forms, depending on the ultimate objectives of the measurements and the type of waste being measured. Methods representative of those available for use on solid, liquid, or gaseous radiological wastes or radiological conditions are discussed below.

SOLIDS-Solid wastes are usually the most difficult to adequately characterize because of their heterogenity. Since drawing conclusions on a large volume of waste from a small sample of that waste requires that the sample be representative of the whole, this aspect warrants special attention. Often the only practical solution to sampling is the application of a statistical approach.

Solid waste release criteria may be in terms of either volume or surface area. Volumetric characteristics can usually be more accurately assessed, if even by destructive examination, than surface characteristics because of often-encountered irregular surfaces with cracks and crevices that hamper both determination of the surface area and the radiological content.

Problem 8-13

Does a flat piece of contaminated steel plate meet the release criterion of 5000 dpm/100 cm^2 if a 70 cm^2 probe with an efficiency of 25%, in contact with the surface, gives 1000 ncpm?

Step 1: Isolate the unknown variable:

$$\frac{1000 ncpm}{0.25 ncpm / dpm} = 4000 \text{ dpm}$$

$$\frac{4000 dpm}{70 cm^2} = \frac{57.1 dpm}{cm^2}$$

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the model setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{57.1 dpm}{cm^2} \bullet 100 \text{ cm}^2 = 5.71 \times 10^3 \text{ dpm}$$

Step 5: Solve for the unknown:

5.71×10^3 dpm>5000 dpm $\therefore$ no release is allowed

Step 6: Conduct a reality check:

Make independent order of magnitude estimate.

Problem 8-14

What is the volume and mass based concentration of 10 Ci of contaminated soil in a 55 gallon drum?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{10Ci}{55gal} \bullet \frac{2.64x10^2 gal}{m^3} \bullet \frac{m^3}{10^6 cm^3} = 4.8 \text{ x } 10^{-5} \frac{Ci}{cm^3}$$

Step 5: Solve for the unknown:

$$4.8 \text{ x } 10^{-5} \frac{Ci}{cm^3} \bullet \frac{cm^3}{2g} = 2.4 \text{ x } 10^{-5} \frac{Ci}{g}$$

Step 6: Conduct a reality check:

Make independent order of magnitude estimate.

LIQUIDS-Liquid wastes can be continuously monitored, proportionally sampled, or grab sampled, depending upon the specific situation and desired use for the information. One approach to continuous monitoring is the submersion of a protected radiation detection chamber into the liquid waste stream. Since the usual units on waste release criteria are activity per unit volume, the detector must be calibrated to read in units of concentration. As for the solids, it is helpful to getting a representative sample, if good mixing has occurred in the waste stream before the measurements are made. The calibration process then entails determining the net counts per minute per unit concentration from using calibrated solutions in the same geometry as the waste stream measurements will be made. A calibration curve can be developed which relates ncpm to concentration, as could have been done for converting ncpm to dpm as discussed in the *Measurement of Radiation* chapter. From continuous monitoring results and waste stream flow data, compliance with both concentration and total release requirements can be assessed. Similar results can be obtained by continuously extracting a small fraction of the waste stream for either on-line or laboratory analysis.

If the concentration and waste stream flow characteristics are fairly constant, statistically valid data can be obtained through a process called ‘proportional sampling”. As this term is normally used, this means a periodic, but discontinuous, sample of volume proportional to the flow volume of the waste stream. Given the assumption of stable waste stream characteristics, compliance with both concentration and total release requirements can be assessed by this means.

A less statistically valid sampling approach for a flowing liquid waste stream is called “grab sampling”. For poorly mixed, unstable waste streams this approach is inappropriate. For well-mixed, stable and contained liquid wastes its validity is much better.

A grab sample is defined as a manually collected single portion of waste. This type of sample reveals the characteristics of the waste at the time the sample was taken. Average characteristics over longer periods of time can be achieved by compositing grab samples taken over a period of time. If frequent grab samples are taken at intervals determined by a waste stream flow rate, then this technique becomes similar to proportional sampling.

Problem 8-15

If a 1% continuous sample of a liquid waste stream, flowing at 10 liters/min is desired, at what rate would the sample need to be withdrawn from the waste stream?

Step 1: Isolate the unknown variable:

Multiplicative model.

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$\frac{10l}{\text{min}} \bullet 10^{-2} = 10^{-1}\,\frac{l}{\text{min}}$$

Step 5: Solve for the unknown:

See Step 4.

Step 6: Conduct reality check.

Make independent order of magnitude estimate.

Problem 8-16

If a submerged radiation detector is calibrated to read 1000 ncpm per µCi/l, at what ncpm level should it be alarmed to indicate a concentration limit of 1 µCi/cm ^{3} ?

Step 1: Isolate the unknown variable:

$$\frac{1000 ncpm}{\mu Ci/l} = \frac{X ncpm}{1 \mu Ci / 10^{-3} l}$$

Step 2: Simplify the equation:

$$\text{X ncpm} = 1000 \bullet \frac{1}{10^{-3}}$$

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

See Step 2.

Step 5: Solve for the unknown:

$$\text{X ncpm} = 10^{6}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

GAS-Air samples can be taken with a variety of objectives in mind. These objectives include: health oriented, environmental source oriented, general movement oriented, public relations oriented, special projects oriented. Each objective can influence the details of how monitoring/sampling are done, how the data are generated and how the data are used. Only representative approaches that might be useful to radiation protection technologists will be included here.

As with liquids, representative samples are obtainable, but not automatic. Assuring a representative, continuous air sample is especially difficult in sampling airborne contaminant particles that are moving at high velocities, as in exhaust ducts. For an air sample to be representative of an effluent stream it must (1) have a gas density equal to the average gas density of the cross section of the effluent stream, (2) contain a pollutant concentration equal to the average concentration of the cross section of the effluent stream, (3) contain a pollutant composition equal to the average composition of the cross section of the effluent stream. The first two of these requirements can be met by proper placement of the sampling probe in the effluent stream.

Sampling at a point of average gas density and average pollutant concentration by itself will not ensure representative sampling. The velocity of the sample as it enters the sampling probe, if it is different from the gas velocity in the stack, can alter the amount of particulate collected.

When gas is drawn into the nozzle with a velocity less than the waste stream velocity, a number of streamlines diverge so that they do not enter the nozzle. The fine particulates are transported along the deflected streamlines, but the larger ones, because of their greater inertia, leave the streamlines during the curvature and enter the nozzle. Hence, the volume of gas extracted will contain more large particles than the same volume contains in the waste stream. Therefore, the particulate matter collected from the gas sample would present a false picture regarding particle size distribution.

When gas is drawn into the nozzle with a velocity greater than the waste stream velocity, a number of streamlines converge and crowd into the nozzle. The fine particulates follow the deflected streamlines, but the larger ones continue along their original straight paths and miss the nozzle completely. Therefore, the gas sample will contain fewer large particles than actually exist in the same volume in the waste stream, and the results of particle size analysis will be false.

When gas is drawn into the nozzle with a velocity equal to the waste stream velocity the streamlines will not be disturbed. As a result, the gas sample will contain the same number of large and small particulates that actually exist in the same volume in the waste stream. Therefore, the gas sample will have the same particulate concentration as the parent stream and the results of particle size analysis will be true. This condition, when the velocity of the gas being withdrawn from the effluent stream equals the velocity of the effluent stream, is essential for the accurate determination of the concentration of particulate matter and particle size distribution in the effluent stream. Such sampling is called "isokinetic sampling" and is determined by the diameter of the sampling probe and the sample withdrawal rate.

Problem 8-17

If the airflow velocity in a duct is 1 m/sec, at what rate would a sample have to be withdrawn from the duct through a 1 cm diameter tube for the sampling to be isokinetic?

Step 1: Isolate the unknown variable:

$$\text{area} = \pi r^2 = \pi \bullet 0.5 = 0.785 \text{ cm}^2$$

$$0.785 \text{ cm}^2 \bullet \frac{m^2}{10^4 cm^2} = 7.85 \text{ x}10^{-5} \text{m}^2$$

$$\text{V [m}^3\text{/sec]} \bullet \frac{1}{7.85x10^{-5} m^2} = 1 \text{ [m/sec]}$$

$$\text{V} = 7.85 \text{ x } 10^{-5} \text{ m}^3\text{/sec}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

See Step 1.

Step 4: Plug in known quantities:

See Step 1.

Step 5: Solve for the unknown:

See Step 1.

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 8-18

If a particulate sample of 24 hours obtained isokinetically at 1 m^3/min yields a result of 10^3 ncpm on a detector with 10% efficiency, what is the particulate concentration in the waste stream?

Step 1: Isolate the unknown variable:

$$\text{CF} = \frac{ncpm}{dpm}$$

$$10^{-1} = \frac{10^3}{dpm} \Rightarrow \text{dpm} = 10^4$$

$$10^4 \text{ dpm} \bullet \frac{pCi \min}{2.2 dis} = 4.55 \text{ x } 10^3 \text{ pCi}$$

Step 2: Simplify the equation:

Multiplicative model.

Step 3: Validate the problem setup:

Will be done while solving the problem.

Step 4: Plug in known quantities:

$$4.55 \text{ x } 10^3 \text{ [pCi]} \bullet \frac{1}{24 hrs} \bullet \frac{hr}{60 \min} \bullet \frac{\min}{1 m^3} \bullet \frac{m^3}{10^6 cm^3} = \text{concentration}$$

Step 5: Solve for the unknown:

$$\text{concentration } = 3.16 \text{ x } 10^{-6} \frac{pCi}{cm^3}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

CHAPTER 9

EXPONENTS AND LOGARITHMS

This chapter presents mathematical basics on exponents and logarithms. The algebra chapter presented earlier in the book (Chapter 2) provided an introduction to exponents and logarithms. Exponents and logarithms represent shorthand mathematical expressions. These shorthand expressions simplify calculations. This chapter will discuss what these shorthand expressions mean and how to readily manipulate the expressions. Included in the discussions will be scientific notation, a fundamental in performing any type of scientific calculation, and natural logarithms, which provided the mathematical basis for a number of health physics applications.

Exponents

Let us look at a multiplicative expression, $y = x \bullet x \bullet x \bullet x$. The expression can be simplified to the expression $y = x^4$. In this shorthand expression, the value 4 is the exponent and x is the base. The two expressions are equivalent. The exponent is 4 in this case since the original expression has four x's multiplied together. When you multiply numbers with the same base, you add the exponents and keep the base the same. Again, our original expression can be written as:

$$y = x^1 \bullet x^1 \bullet x^1 \bullet x^1$$

To arrive at the exponential expression, you keep the same base, x, and add the exponents, 1+1+1+1, or 4.

Conversely, when you have an expression that divides two numbers, you subtract the exponents. For example, take the following expression:

$$y = \frac{x^5}{x^2} = x^5 - x^2 = x^{5-2} = x^3$$

Since the two exponential expressions have the same base, x, then the simplified expression is x^{5-2}, or x^3.

If you have an exponential expression, raised to a power, then you multiply the exponents. For example, let's look at the expression representing the square of the square root of x:

$$y = (x^{1/2})^2 = x^{½ \bullet 2} = x^1$$

In this case, the expression is simplified to $y = x^1$ (or simply x), since the product of ½ times 2 is 1. If you have a multiplicative expression raised to a power, then all the factors are raised to that power. For example:

$$y = (a^2 \bullet x^{1/3})^2$$

This expression simplifies to $y = a^{2 \bullet 2} \bullet x^{1/3 \bullet 2}$ or, $y = a^4 \bullet x^{2/3}$.

Examples

Simplify the following expressions using exponential notation:

1. $y = a \bullet a^2 \bullet a \bullet a \bullet a$
2. $y = x + x + x$
3. $y = (b^3)^{1/2}$
4. $y = x^2/x^3$
5. $y = x^4/a^2$

Step 1: Isolate the unknowns, or variables, on one side of the equation and the constants on the other side of the equation.

The expressions are already in this form. Although there are variables on both sides of the equations, we are looking for a simplified expression for y.

$y = a \bullet a^2 \bullet a \bullet a \bullet a$
$y = x + x + x$
$y = (b^3)^{1/2}$
$y = x^2/x^3$
$y = x^4/a^2$

Step 2: Simplify the equation to the form $x = C$, where x is the unknown and C is a constant. C is then the solution to the unknown.

$y = a^{1+2+1+1+1} = a^6$
$y = x + x + x = 3x$
$y = (b^3)^{1/2} = b^{3/2}$
$y = x^2/x^3 = x^{-1}$
$y = x^4/a^2 = x^4/a^2$

Step 3: Verify the solution by putting the solution for x into the *original* equation to check to see if the equality is correct.

We do not have values for our dependent variable (that is the variables a, x, and b) to plug into the equation. To verify the solutions, you could select values for the dependent variables. For example, let $a = 2$ for the first expression. You would then get $y = 2 \bullet 2^2 \bullet 2 \bullet 2 \bullet 2$, or, $2 \bullet 4 \bullet 2 \bullet 2 \bullet 2$, which equals 64. 2^6 is also 64.✓

Note that for #2, the answer is $3x$, not x^3. The x's are added together in the problem, not multiplied together, so the exponents are not added. Also, in #5, the bases in the numerator (x) and denominator (a) are different, so the exponents are not subtracted.

Scientific Notation

A special application of exponents is in scientific notation. Scientific notation is a shorthand way of writing large number using 10 as a base. Here are values of base ten raised to whole number exponents:

Number	Exponent of 10
1	10^0
10	10^1
100	10^2
1,000	10^3
10,000	10^4
100,000	10^5
1,000,000	10^6
0.1	10^{-1}
0.01	10^{-2}
0.001	10^{-3}
0.0001	10^{-4}
0.00001	10^{-5}
0.000001	10^{-6}

If you follow the pattern in the table above, you see that for positive powers of 10, the exponent value is equal to the number of digits to the right of the first number. For example, the number 10,000 has four zeros and the exponential of 10 is 10^4. For fractions of 10, the exponent value is equal to the negative of the number of digit after the decimal place (including the 1). For example, 0.001 has three digits after the decimal point and the exponential of 10 is 10^{-3}.

A number is in scientific notation when it is expressed as the product of a decimal number between 1 and 10 and some integer power of 10.

The steps for converting a number to scientific notation are as follows:

Step 1: Place the decimal immediately to the right of the left-most non-zero number.

Step 2: Count the number of digits between the old and new decimal point. This is the exponent of 10.

Step 3: If the decimal is shifted to the left, the exponent is positive. If the decimal is shifted to the right, the exponent is negative.

Step 4: Write the expression a x 10^n, where a is the number with the decimal immediately to the right of the left-most non-zero number (Step 1) and n is the number of digits the decimal was shifted (Step 2), with the appropriate sign (Step 3).

For our first example, let's take 4,611,000:

Step 1: Place the decimal immediately to the right of the left-most non-zero number: **4.611000**

Step 2: Count the number of digits between the old and new decimal point. **6 (remember that 4,611,000 is the same as 4,611,000., so you move the decimal from the end six places to after the 4.)**

Step 3: If the decimal is shifted to the left, the exponent is positive. If the decimal is shifted to the right, the exponent is negative. **The exponent is positive.**

Step 4: Write the expression a x 10^n, where a is the number with the decimal immediately to the right of the left-most non-zero number (Step 1) and n is the number of digits the decimal was shifted (Step 2), with the appropriate sign (Step 3).
4.611 x 10^6 (you can ignore the last three zeros).

For our next example, let's take 0.00222:

Step 1: Place the decimal immediately to the right of the left-most non-zero number: **2.22**

Step 2: Count the number of digits between the old and new decimal point. **3**

Step 3: If the decimal is shifted to the left, the exponent is positive. If the decimal is shifted to the right, the exponent is negative. **The exponent is negative.**

Step 4: Write the expression a x 10^n, where a is the number with the decimal immediately to the right of the left-most non-zero number (Step 1) and n is the number of digits the decimal was shifted (Step 2), with the appropriate sign (Step 3).
2.22 x 10^{-3}

The table below lists values in the first column and their corresponding scientific notation in the second column. You can use the values in the table to practice converting numbers into scientific notation. Also, most calculators can display numbers in scientific notation or decimal forms by toggling a setting. Refer to your owner's manual to determine if your calculator can be set to express numbers in scientific notation.

Number	Scientific Notation
131	1.31×10^2
11.134	1.1134×10^1
134,443.3	1.344433×10^5
20,000,000	$2. \times 10^7$
456.08	4.5608×10^2
6,023,000	6.023×10^6
0.0034	3.4×10^{-3}
0.0000555	5.55×10^{-5}
0.009	$9. \times 10^{-3}$
0.56	5.6×10^{-1}
0.000088	8.8×10^{-5}
4	$4. \times 10^0$
2332	2.332×10^3

Logarithms

Any number can be expressed as any non-zero number raised to an exponent. The exponent and the base number define a logarithm. For example:

1. $4^2 = 16$
2. $2^4 = 16$
3. $16^1 = 16$
4. $10^{1.2} = 16$
5. $3^{2.524} = 16$
6. $1.7411^5 = 16$
7. $2.718281828459042^{2.7726} = 16$

In example #1, the base is 4 and the exponent is 2. In example #2, the base is 2 and the exponent is 4. In example #3, the base is 16 and the exponent is 1. In example #4, the base is 10 and the exponent is 1.2. In example #5, the base is 3 and the exponent is 2.524. In example #6, the base is 1.7411 and the exponent is 5. Finally, in example #7, the base is 2.718281828459042 (you will see later in the chapter that this number has some significance!) and the exponent is 2.7726.

A logarithm is the exponent, given a specific base. Using our first example above, the logarithm of 16 to the base 4 is 2. This expression is written as $\log_4 16 = 2$. The following lists the expressions for the other six examples:

2. $\log_2 16 = 4$
3. $\log_{16} 16 = 1$
4. $\log_{10} 16 = 1.2$
5. $\log_3 16 = 2.524$
6. $\log_{1.7411} 16 = 5$
7. $\log_{2.718281828459042} 16 = 2.7726$

Example #3 shows that the log of any number to the base of that number is 1. This rule makes sense, since we know that any number raised to the power of 1 equals that same number. Example #4 also represents a special case. Base 10 logarithms are commonly used to express numbers. Often, the base is not written in base 10 logarithms. In this case, example #4 would become log 16 = 1.2.

Many scientific calculators have a base 10 logarithm feature. If you enter 16 into your calculator and it the "log" button, the result should be 1.2. These calculators also have an anti-logarithm feature. An anti-logarithm answers the question "What is the result of raising a given base to an exponent?" In example #4, the anti-logarithm answers the question "What is the result of raising base 10 to the 1.2 power. If you enter 1.2 into your calculator and press the "10^x" button (or maybe the "INV" and then "log" buttons), you should get the result of 16. Please consult the owner's manual for your calculator to determine the base 10 logarithm function.

Natural Logarithms

As we hinted at above, example # 7 is also a special case. In science and engineering, the value 2.718281828459042 is a common logarithm base. To avoid having to write out this number for each expression, the symbol *e* has been chosen to represent the number. Logarithms to the base *e* are called natural logarithms and $\log_e$ is further abbreviated as ln.

As with base 10 logarithms, most scientific calculators have a natural log feature. If you enter the value 16 and press the "ln" key, the result should be 2.77 (our answer in example #7). Also, scientific calculators can take anti-natural logarithms. If you enter 2.7726 and press the "e^x" key, you will get 16. Please consult the owner's manual for your calculator to determine the natural logarithm function.

As you will see in chapters in this book on radioactive decay (Chapter 10) and interaction of radiation and matter (Chapter 11), natural logarithms play an integral part in solving problems. In most cases, it is the anti-natural logarithm that is important. In other words, you will need to answer the question "What is the result of raising *e* to the power of …?" In these problems, the natural logarithm base will be raised to the power of the product of the decay constant and a time period or the product of an attenuation coefficient and a material thickness.

CHAPTER 10

RADIOACTIVITY AND RADIOACTIVE DECAY

As with most things in nature, atoms want to be in their most stable state. An unstable atom changes its nuclear state through radioactive transformation, or decay. This transformation takes place when an atom converts neutrons to protons, or protons to neutrons, and then releases the excess mass or energy from the nucleus. This emission of particles or energy from the nucleus is called *radiation*. Radiation can be in the form of particles or electromagnetic energy waves. These emissions occur randomly as each atom tries to achieve a more stable state. The property of certain nuclides to spontaneously emit radiation is called *radioactivity*. This emission is called radioactive decay.

Following a transformation the nucleus is usually more stable than it was, but it may not be completely stable. So, another transformation will take place in which the nucleus will again emit radiation. The amount of energy given off and the type of emission that occurs will depend on the configuration of the nucleus immediately before a specific transformation occurs. The original configuration of the nucleus (that is, before the decay) is called the *parent* and the configuration after the decay is called the *daughter*. If the daughter radionuclide is unstable, or radioactive, then it will decay to a different daughter product. This continuing decay of daughter products results in a radioactive decay chain, or series.

Radioactive decay of a radionuclide is characterized by three things: the types of radiation emitted, the energies of the radiation, and the rate at which the nuclide decays. These three characteristics, when grouped together, comprise a unique fingerprint for the radionuclide.

Types of Radiation

Radionuclides transform by a variety of methods, depending on their degree of instability and configuration of the nucleus. These decay methods include alpha emission, beta emission, gamma ray emission, positron emission, and orbital electron capture.

Alpha Emission. In alpha radiation emission, an unstable radionuclide emits an alpha particle, a highly energetic helium nucleus. The alpha particle is comprised of two protons and two neutrons. The daughter product has an atomic mass number of four less than the parent's atomic mass number and atomic number (number of protons) two less than the parent's atomic number. For example:

$$^{226}_{88}Ra \rightarrow \; ^{4}_{2}\alpha \; + \; ^{222}_{86}Rn$$

Alpha particles are released from radionuclides that have low neutron to proton ratios. Alpha particles are energetic. Most alpha particles have energies greater than 3.8 MeV. Alpha particles are emitted in discrete energies. It is possible to identify and quantify an alpha-emitting radionuclide by measuring the energy of the emitted alpha particles.

Alpha particles cause dense ionizations as they move through mater. As such, their energy is relinquished rapidly and they travel only short distances. A piece of paper can stop an alpha particle.

Beta Emission. A beta particle is an electron that is emitted from the nucleus of a radionuclide. Beta particles are emitted from radionuclides with an excess number of neutrons. A neutron is converted to a proton and an electron and the electron is emitted from the nucleus. The atomic weight of the daughter product is the same as the parent, but since a proton is formed from a neutron, the atomic number is increased by one. For example:

$$ {}^{3}_{1}H \rightarrow {}^{0}_{-1}e + {}^{3}_{2}He $$

The radionuclide will emit a beta particle with a continuous range of energies for 0 to a maximum beta energy. The maximum energy depends on the radionuclide. The average energy of the emitted beta particle is approximately one-third the maximum energy. The radionuclide will also emit neutrinos, particles with no mass, to make up for the additional energy (energy of the emitted beta + the energy of the neutrinos equals the maximum beta energy).

Beta particles are more penetrating then alpha particles. They are stopped by a thick sheet of plastic or thin sheet of metal.

Gamma Ray Emission. Gamma rays are electromagnetic energy originating from the nucleus of the radionuclide. Gamma rays have no mass and no charge. When emitted, they do not alter the atomic weight or atomic number of the radionuclide. Gamma radiation is emitted when a radionuclide must release excess energy, yet has a stable ratio of protons to neutrons. For example, many alpha emitting radionuclides emit particles of different energies. The radionuclide will emit a gamma ray to release any excess energy not carried away by the alpha particle. Likewise, many beta emitting radionuclides also emit gamma rays. These gamma rays are emitted at discrete energies. Gamma spectroscopy systems can measure the energy of the emitted gamma ray and determine what radionuclides are present.

Gamma rays are the most penetrating ionizing radiation. It takes materials such as lead, concrete, and water to stop gamma rays.

Positron Emission and Orbital Electron Capture. Positron emission is similar to beta emission. Instead of releasing a negatively charged electron, a neutron-deficient nucleus emits a positively-charged electron. Unlike negatively-charged electrons, positrons do not exist in nature. Once released, they interact with negatively-charged electrons and are annihilated. This annihilation is accompanied by the formation of two 0.51 MeV gamma rays. Alternatively, this neutron-deficient radionuclide may capture an orbital electron. This orbital electron is combined with a proton to convert that proton to a neutron.

Radioactive Decay

The process in which a radionuclide emits one or more of the types of radiation discussed above is radioactive transformation or radioactive decay. Radioactive decay is a random event. It is not known when any specific atom of a radionuclide will decay. As with other random events, all we can do is establish properties of a population of atoms. To illustrate this point, let's look at a gram of radium-226 in a container in front of us. Radium-226 has the following properties:

- Half-live of 1,602 years
- Radiation emitted includes alpha particles and gamma radiation
- Atomic weight of 226, or 226 grams in one mole of Ra-226
- Daughter product is radon-222, which is radioactive

Before we go any further, we need to look at a basic concept for high school chemistry. One mole of any element has 6.0247×10^{23} atoms, Avogadro's number. In the case of Ra-226, there are 6.0247×10^{23} atoms in 226 grams of Ra-226. To find the number of atoms in one gram of Ra-226, we divide Avogadro's number by 226. The result is that there are 2.67×10^{21} atoms in our container of Ra-226 when we start. Now, as we mentioned earlier, we cannot predict which of those 2.67×10^{21} atoms will decay, or transform into Rn-222. All we know with any certainty is that half of all the Ra-226 present in our container will transform in 1,602 years. The half-life of Ra-226 is 1,602 years and as the name implies is the length of time it takes for one half of the atoms present to undergo the radioactive decay process.

Different radionuclides decay at different rates. These decay rates depend on the half-life of the radionuclide. In our example above, the Ra-226 has a half-life of 1,602 years while the its daughter product, Rn-222, decays with a half-life of 3.05 days. The rate of decay of a radioactive substance constitutes the quantity of radioactivity, or *activity*, in that substance. The definition of activity refers to the number of transformations per unit time. The special unit for activity is the Curie (Ci). One Ci is equal to 37 billion (3.7×10^{10}) transformations per second. For radioactive material, it is more common to talk in terms of activity than in terms of mass. You will more often deal with 1 microcurie of Co-60 instead of 1 microgram of Co-60. To determine the activity associated with a given number of atoms of a radioactive element, the following formula is use:

$$A = \lambda \bullet N \bullet C_1 \qquad \text{Equation 10-1}$$

where,

A = activity, in curies
λ = radioactive decay constant
N = number of atoms of the radioactive element present
C_1 = constant to convert to curies, 2.7×10^{-11} curies per transformation

The radioactive decay constant, λ, is the ratio of the natural logarithm of 2 to the half-life of the radionuclide:

$$\lambda = \frac{\ln 2}{T_{1/2}} \qquad \text{Equation 10-2}$$

The natural logarithm of 2 can be approximated at 0.693. To verify this approximation, you can enter 2 in your calculator and press the "ln" function key.

Problem 10-1

What is the activity in 1 gram of Ra-226?

Step 1: Isolate the unknown variable:

$$A = \lambda \bullet N \bullet C_1$$
$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Step 2: Simplify the equation:

Substitute the expression for the decay constant onto the equation:

$$A = \frac{\ln 2}{T_{1/2}} \bullet N \bullet C_1$$

Step 3: Validate the problem set-up:

$$A\ (curies) = \frac{\ln 2}{T_{1/2}(\text{sec})} \bullet N\ (atoms) \bullet C_1\ (curies\ per\ transformation) \checkmark$$

At first glance, it may appear that the units in the above equation do not cancel out. However, the N in the equation represents the number of atoms available for transformation and these atoms cancel with the "transformation" unit in the constant. Also note that the half-life is in units of seconds.

1,602 years • 365 days per year • 24 hours per day • 3600 seconds per hour = 5.05×10^{10} seconds ⇒ the half-life of Ra-226 in terms of seconds.

Step 4: Plug in known quantities:

$$A\ (curies) = \frac{\ln 2}{5.05\ x\ 10^{10}(\text{sec})} \bullet 2.67\ x\ 10^{21}\ (atoms) \bullet 2.7\ x\ 10^{-11}\ (curies\ per\ transformation)$$

Step 5: Solve for the unknown:

$A = 0.99$ Ci

Step 6: Conduct reality check:

The unit curies was defined as the number of transformations per second in one gram of Ra-226, or one gram of Ra-226 has an activity of 1 Ci. ✓

The SI derived unit of activity is the *becquerel* (Bq) and is that quantity of radioactive material in which one atom is transformed per second or undergoes one disintegration per second (1 dps). Since the becquerel is a rather small unit, metric prefixes are often applied to aid in designating larger amounts of activity. For example, an activity of one million transformations per second is one megabecquerel (MBq). An activity of one billion transformations per second is one gigabecquerel (GBq).

Specific activity is defined as the activity per unit mass of a radioactive substance and is reported in units such as curies per gram (Ci/g) or becquerels per kilogram (Bq/kg). As we showed in the example above, one gram of Ra-226 has an activity of 1 Ci. In other words, the specific activity of Ra-226 would be 1 Ci/g. The following expression can be used to determine the specific activity of any radionuclide.

$$SA = \frac{\lambda \bullet N_{Avogadro}}{AW} \bullet C_1 \qquad \text{Equation 10-3}$$

where,

SA = the specific activity, curies per gram

$N_{Avogadro}$ = Avogadro's number, 6.0247×10^{23} atoms

AW = Atomic weight, in grams

λ = radioactive decay constant

C_1 = constant to convert to curies, 2.7×10^{-11} curies per transformation

Problem 10-2

What is the specific activity of Rn-220? U-238?, H-3?

Step 1: Isolate the unknown variable:

$$SA = \frac{\lambda \bullet N_{Avogadro}}{AW} \bullet C_1$$

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Step 2: Simplify the equation:

Substitute the expression for the decay constant onto the equation:

$$SA = \frac{\frac{\ln 2}{T_{1/2}} \bullet N_{Avogadro}}{AW} \bullet C_1$$

Step 3: Validate the problem set-up:

$$SA\ (Ci/g) = \frac{\frac{\ln 2}{T_{1/2}(\text{sec})} \bullet N_{Avogadro}\,(atoms)}{AW\ (grams)} \bullet C_1\ (transformation/\text{sec}) \checkmark$$

As with our first example, the half-life is in units of seconds.

Step 4: Plug in known quantities:

Half-lives:
Rn-220: 3.05 days • 24 hours/day • 3600 sec/hr = 2.64 x 10^5 sec
U-238: 4.5 x 10^9 years • 365 days • 24 hours/day • 3600 sec/hr = 1.42 x 10^{17} sec
N-16: 7.2 sec

<u>Rn-220</u>

$$SA\ (Ci/g) = \frac{\frac{\ln 2}{2.64 x 10^5(\text{sec})} \bullet 6.0247 x 10^{23}\,(atoms)}{220\ (grams)} \bullet 2.7 \text{ x } 10^{-11}\ \ (transformation/\text{sec})$$

<u>U-238</u>

$$SA\ (Ci/g) = \frac{\frac{\ln 2}{1.42 x 10^{17}(\text{sec})} \bullet 6.0247 x 10^{23}\,(atoms)}{238\ (grams)} \bullet 2.7 \text{ x } 10^{-11}\ \ (transformation/\text{sec})$$

N-16

$$SA\,(Ci/g) \;=\; \frac{\frac{\ln 2}{7.2(\text{sec})} \bullet 6.0247x10^{23}\,(atoms)}{16\,(grams)} \bullet 2.7 \text{ x } 10^{-11}\ \ (transformation/\text{sec})$$

Step 5: Solve for the unknown:

Rn-220: SA = 1.94 x 10^5 Ci/g
U-238: SA = 3.34 x 10^{-7} Ci/g
N-16: SA = 9.79 x 10^{10} Ci/g

Step 6: Conduct reality check:

The specific activity gets larger as the half-life gets shorter. We see in are example that the specific activity of U-238 is very small and its half-life is very long. The converse is true for Rn-220 and N-16. ✓

There is also a quick way to estimate the specific activity for any radionuclide, using a ratio to Ra-226. This ratioing works since the specific activity of Ra-226 is unity. The expression is:

$$SA \;=\; \frac{Half-life,\ Ra-226 \bullet Atomic\ Weight,\ Ra-226}{Half-life,\ radionuclide \bullet Atomic\ Weight,\ radionuclide}$$

The Radioactive Decay Curve

The expression above for activity, $A \;=\; \lambda \bullet N \bullet C_1$, gives the activity associated with a given number of atoms of a radionuclide present. Of course, the atoms of the radionuclide are constantly undergoing radioactive decay, so the number of atoms present constantly changes. Sometimes it is important to know what activity will be present as a function of time.

An easy way to show how the activity of a radionuclide changes with time is to plot the activity over time on a graph. We have already said that over one half-life, the activity decreases by ½. The activity after two half-lives would be ¼, and so on. This relationship yields the following relationship for the activity after "n" half-lives given a known activity at time zero:

This relationship yields the following relationship for the activity after "n" half-lives given a known activity at time zero:

$$\frac{A_n}{A_0} \;=\; \frac{1}{2^n}$$ Equation 10-4

Exhibit 10-1 shows the plot of relative activity as a function of the number of half-lives. As you can see from the plot, the activity decreases rapidly at first and then tails off near zero. Although difficult to see from Exhibit 10-1, the activity never gets to zero. It is like if you stand at one side of a room and walk half the distance to the opposite wall, and then half the distance again, and half the distance again, you never get to the wall. There is always some space of which you can take half the distance.

As can be seen with Exhibit 10-1, it is difficult to distinguish the activity after seven half-lives. A more convenient way to plot the decay curve is shown in Exhibit 10-2. The y-axis scale has been converted to a logarithmic scale. The x-axis is still a linear scale. This type of plot is called a semi-log plot and you have seen examples of these plots earlier in the book. As can be seen in Exhibit 10-2, the semi-log plot of relative activity is a straight line. We can also determine the relative activity present after 7, 8, 9, or 10 half-lives.

Exhibit 10-2 tells us two things. First, the change in activity over time is linear when plotted on a semi-log plot. From this fact, we know that the equation for activity at any time "t", is given by an exponential formula. The exact formula is:

$$A_t = A_0 \bullet e^{-\lambda \bullet t} \qquad \text{Equation 10-5}$$

A_t is the activity at any time t, given an activity at time zero of A_0. The decay constant, λ, is the same constant as we discussed earlier, $\lambda = \frac{\ln 2}{T_{1/2}}$.

Second, a rule of thumb can be developed based on the plot. Although we have said that the activity never gets to zero, we can say when the radioactivity is approximately zero. From Exhibit 10-2, we see that at seven half-lives, there is less than one percent of the activity remaining and after ten half-lives there is one-tenth of one percent of the original activity remaining. We can say that after between seven and ten half-lives, the activity is essentially zero. Caution must be taken for very large sources of radioactivity. One percent or even one-tenth of one percent still could be a large amount of activity if you begin with a very large amount of activity.

Problem 10-3

Plot the activity, as a function of time in years, of H-3, Mn-54, and Ni-63. The activity at time zero for each of these radionuclides is 1 curie. These radionuclides have the following half-lives:

H-3:	12.3 years
Mn-54:	303 days
Ni-63:	92 years

Exhibit 10-1.
Radioactive Decay Curve

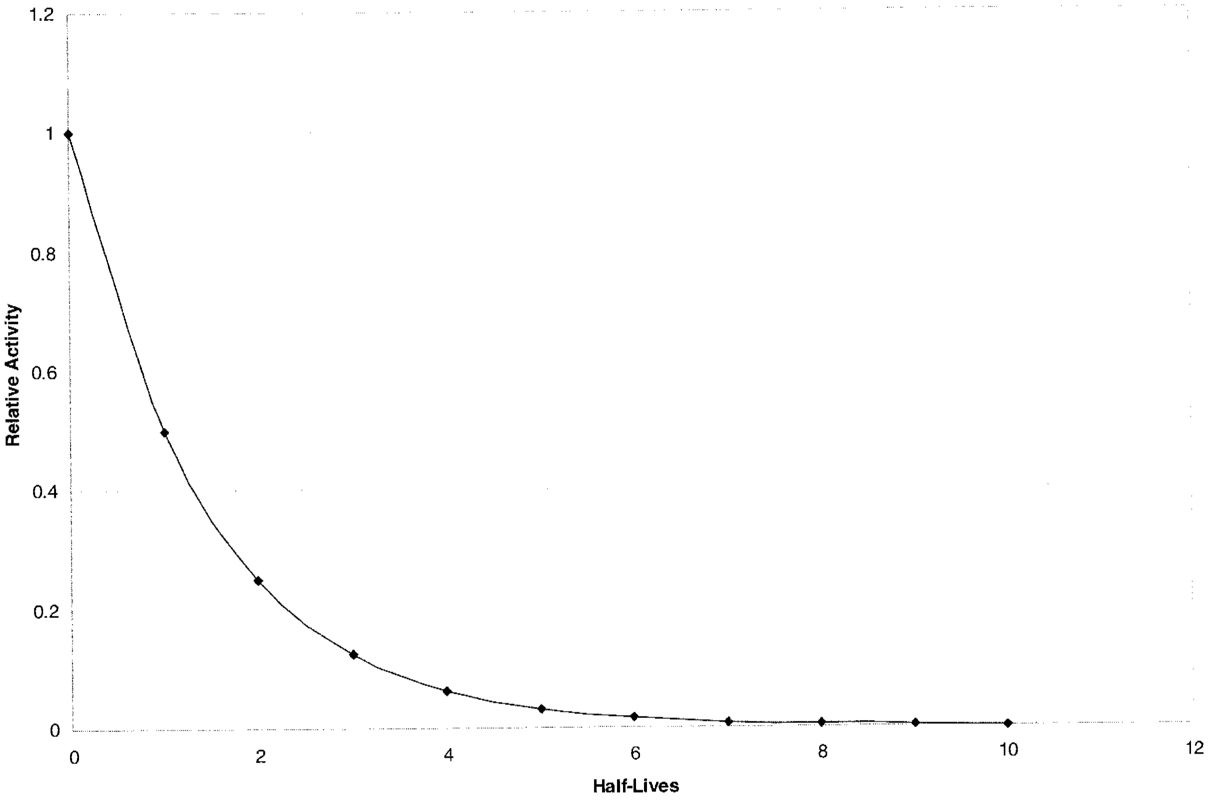
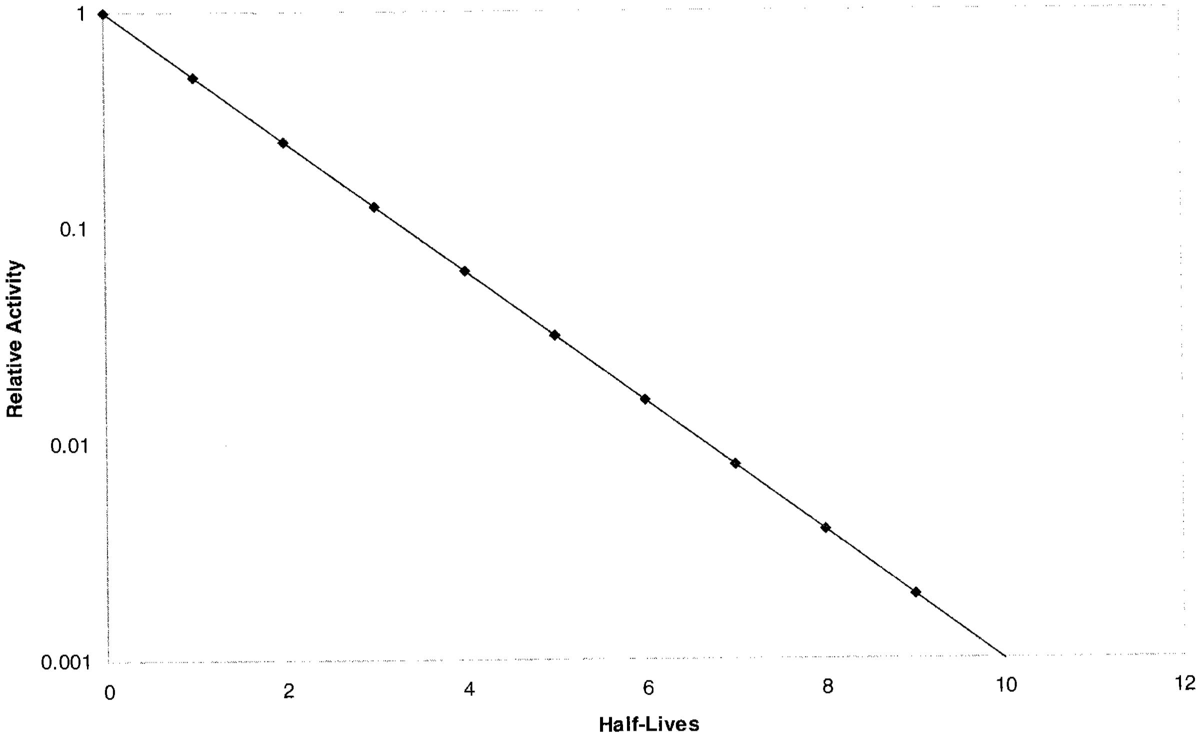


Exhibit 10-2.
Radioactive Decay Curve, Semi-Log Plot



Step 1: Isolate the unknown variable:

$$A_t = A_0 \bullet e^{-\lambda \bullet t}$$

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Step 2: Simplify the equation:

Substitute the expression for the decay constant onto the equation:

$$A_t = A_0 \bullet e^{-\frac{\ln 2}{T_{1/2}} \bullet t}$$

Step 3: Validate the problem set-up:

$$A_t\ (curies) = A_0\ (curies) \bullet e^{-\frac{\ln 2}{T_{1/2}\ (years)} \bullet t\ (years)} \quad \checkmark$$

Note that we need are half-lives in years. To convert the Mn-54 half-life:

Mn-54: 303 days • 1year per 365 days = 0.83 years.

Step 4: Plug in known quantities:

<u>H-3</u>

$$A_t\ (curies) = 1\ (curie) \bullet e^{-\frac{\ln 2}{12.3\ (years)} \bullet t\ (years)}$$

<u>Mn-54</u>

$$A_t\ (curies) = 1\ (curie) \bullet e^{-\frac{\ln 2}{0.83\ (years)} \bullet t\ (years)}$$

<u>Ni-63</u>

$$A_t\ (curies) = 1\ (curie) \bullet e^{-\frac{\ln 2}{92\ (years)} \bullet t\ (years)}$$

Step 5: Solve for the unknown:

The following table presents the activity for given times between 0 and 100 years.

Time	H-3	Mn-54	Ni-63
0	1	1	1
10	0.569262	0.000237	0.927441
20	0.324059	5.6E-08	0.860147
30	0.184475	1.32E-11	0.797735
40	0.105014	3.13E-15	0.739853
50	0.059781	7.4E-19	0.68617
60	0.034031	1.75E-22	0.636382
70	0.019372	4.14E-26	0.590207
80	0.011028	9.8E-30	0.547382
90	0.006278	2.32E-33	0.507664
100	0.003574	5.48E-37	0.470829

Step 6: Conduct reality check:

Exhibit 10-3 presents the semi-log plots for the three radionuclides. You can see from the plot that the three decay plots have very different slopes. The slop of each plot is a function of the half-life. The shorter the half-life, the steeper the slope of the plot. ✓

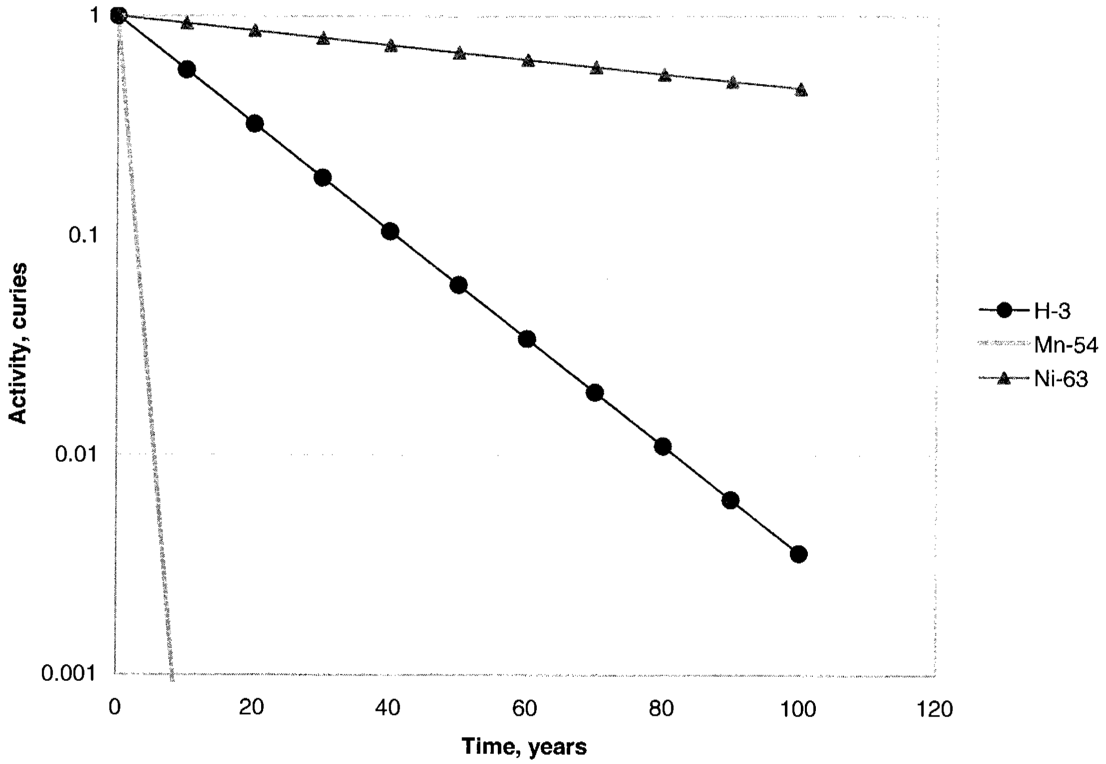
Decay Series

As we touched upon earlier, in a radioactive decay series, the decay of the parent nuclide produces a daughter product that is radioactive. The daughter nuclide also produces radiation when it decays, as does each successive daughter in the chain until a stable isotope is produced. This series of transformations is known as a decay series, or decay chain. The activity contributed from the parent versus the daughters will vary depending on the half-life of the parent and the half-lives of the daughters. When the amount of activity being produced is the same as the amount that is decaying, the chain has reached equilibrium. There are two types of equilibrium, depending on how the half-life of the daughter compares to the half-life of the parent: *secular equilibrium* and *transient equilibrium.*

In secular equilibrium the half-life of the parent is very much longer than the half-life of the daughter. When in equilibrium, the activity of the daughter is equal to the activity of the parent. Equilibrium is reached in approximately seven half-lives of the daughter. The most common illustrations of secular equilibrium are the natural occurring decay series. There are four natural decay series. The four series are headed by Th-232, U-235, U-238, and Pu-241. The fourth series, the one headed by Pu-241, is no longer present in nature. The other three series have a number of similarities, other than still existing today in nature. All three series have isotopes of radium and radon in the chain. Also, all the series end in a stable form of lead.

Exhibit 10-3

Radioactive Decay Plots



In transient equilibrium the half-life of the parent is longer than that of the daughter, but not very much longer. After the daughter activity builds up and reaches equilibrium, it decays with the same rate of decay as the parent. In other words, the effective half-life of the daughter is the same as that of the parent.

When the half-life of the parent is shorter than that of the daughter, the two never reach equilibrium.

CHAPTER 11

INTERACTION OF RADIATION AND MATTER

Most of the mathematical concepts required to solve radiation interaction problems were presented in the last chapter on radioactive decay, since both types of problems require the use of the same equations with the variables being redefined slightly. However, several new and/or often misused definitions are involved in radiation interactions, so we will start with them.

First, there is often confusion in the use of the terms "attenuation" and "absorption". When properly used, "attenuation" refers to the reduction in the intensity of a beam of radiation by a combination of absorption and scattering. "Absorption" is the process by which radiation imparts some or all of its energy to materials through which it passes. "Scattering" is defined as a change in direction as a result of a collision or interaction. The importance of these distinctions will become more apparent as the various interaction processes are discussed.

Gamma Attenuation

The equation that is most often associated with the attenuation of gamma photons is

$$I = I_o e^{-\mu x} \qquad \text{Equation 11-1}$$

where,

I = final gamma intensity

I_o = initial gamma intensity

e = base of natural logarithm

μ = linear attenuation coefficient [cm^{-1}]

x = absorber thickness [cm]

Strictly speaking, this equation is only valid for situations where any interaction of the incident radiation and the absorbing medium removes that photon from the beam, whether the interaction is absorption or scattering. Geometries for which this condition is true are often referred to as "narrow beam" geometries. Conditions where this is not true are referred to as "broad beam " geometries. The latter geometries allow scattered photons to reenter the primary beam, where the former geometries preclude this by the use of shielding collimators or the purity of the nature of the absorbing interaction.

Equation 11-1 is often seen written in other forms like:

$$I = I_o e^{-(\mu/\rho)(\rho x)} \qquad \text{Equation 11-2}$$

where all of the variables are defined as in Equation 11-1 and

ρ = density of absorbing material.

The advantage of using this form of the equation is that the μ/ρ term, called the "mass attenuation coefficient" is relatively independent of the absorbing material for photon energies between about 0.7 and 5 MeV. Some μ/ρ data are tabulated in Exhibit 11-1.

Exhibit 11-1.
Mass Attenuation Coefficients (μ/ρ)

Photon Energy [keV]	Air	Water	Concrete	Iron	Lead
800	0.0707	0.0786	0.0709	0.0669	0.0885
1000	0.0636	0.0707	0.0637	0.0599	0.0708
1500	0.0518	0.0575	0.0519	0.0488	0.0517
2000	0.0445	0.0494	0.0448	0.0362	0.0455

Both of these equation formulations require narrow beam geometries to be truly applicable. However, they are often incorrectly applied to all geometries, but are often good enough for first approximations in fieldwork. The analogy of this requirement, in the context of radioactive decay, it is assumed that once an atom decays, it no longer is radioactive. If this is not the case, we adjust the decay equation to reflect the ingrowth of a radioactive daughter. How "broad beam" geometries are correctly handled is radiation protection problems will be discussed later in this chapter.

Problem 11-1

What exposure rate of Cs-137 photons is transmitted through a 1 cm Pb shield, if the incident rate is 1 R/hr?

Step 1: Isolate the unknown variable:

$$I = I_0 e^{-(\mu/\rho)(\rho x)}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

R/hr = R/hr

Step 4: Plug in known quantities:

$I = 1 \bullet e^{-(0.0885)(11.35)(1)}$ E = 0.66 MeV and $\rho = 11.35$ g/cm ^{3}

Step 5: Solve for the unknown:

I = 0.366 R/hr

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-2

What is the incident flux of Co-60 photons on a 1 inch thick iron shield if 10^6 photons/cm ^{2} /sec emerge from the shield, given Co-60 photon energy (total) is ≈ 2500 keV?

Step 1: Isolate the unknown variable:

$$I_0 = \frac{I}{e^{-(\mu/\rho)(\rho x)}}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

photons/cm ^{2} /sec = photons/cm ^{2} /sec

Step 4: Plug in known quantities:

$$I_0 = \frac{10^6}{e^{-(0.0544)(7.86)(2.54)}}$$ Interpolate $\mu/\rho = 0.0544$ from Exhibit 11-1

Step 5: Solve for the unknown:

$$I_0 = \frac{10^6}{0.338} = 2.96 \times 10^6 \text{ photons/cm}^2\text{/sec}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-3

What is the ratio of transmitted to incident photon dose rate for 300 keV photons and 50 cm water shield?

Step 1: Isolate the unknown variable:

$$\frac{I}{I_0} = e^{-(\mu/\rho)(\rho x)}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{I}{I_0} = e^{-(0.119)(1)(50)}$$

Step 5: Solve for the unknown:

$$\frac{I}{I_0} = 2.61 \text{ x } 10^{-3}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-4

What is the μ/ρ of a material with a density 2.7 g/cm^3, that exhibits a tenth value layer of 10 cm?

Step 1: Isolate the unknown variable:

$$\mu/\rho = -\frac{\ln(\frac{I}{I_0})}{\rho x}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\text{cm}^2/\text{g} = \frac{1}{(g/cm^3)(cm)}$$

Step 4: Plug in known quantities:

$$\mu/\rho = -\frac{\ln(1/10)}{(2.7)(10)} = -\frac{-2.3}{(2.7)(10)}$$

Step 5: Solve for the unknown:

$$\mu/\rho = 0.0852 \text{ cm}^2/\text{gram}$$

Step 6: Conduct reality check:

Coefficient is right in range of published values.

There are three primary reactions through which gamma photons interact with matter. At low energies ($E < 0.1$MeV) most interactions are via a process called the "Photoelectric Effect". In this process the photon is completely absorbed when it gives up its entire energy to an orbital electron of the absorbing medium, thus releasing the electron from its nucleus (ionization) and giving the electron some kinetic energy. The cross section (or probability) for this process is proportional to $Z^4\ \lambda^3$ or Z^5 of the absorbing material.

For photons with $0.1 < E < 5$ MeV, most interactions are via a process called "Compton Scattering". In this process the incident photon has too much energy to be taken away by the electron only, so a secondary photon is scattered, carrying away the excess energy (a photon of lesser energy). The cross section for this process is proportional to Z of the absorbing material.

Beginning at photon energies of 1.02 MeV, a process called "Pair Production" can occur. In this process, the high-energy photon interacts with the nucleus of an atom of the absorbing material and an electron/positron pair is created. This process represents the creation of matter from energy as predicted by Einstein's $E = mc^2$ and is the opposite of the process of antiparticles annihilating one another, resulting in the emission of two photons (both with energy of 0.511 MeV). The cross section for this process is proportional to Z^2 and becomes predominant at E>5 MeV.

It can be seen that only the photoelectric effect, of the three interaction modes, meets the requirements for narrow beam geometry without the use of collimators, etc., or without modification of the equation to accommodate scattering (in the case of Compton scattering) or the creation of bremsstrahlung and annihilation radiation (in the case of Pair Production). The modification of the most often used formulation of the attenuation equation (Equation 11-2) is the insertion of a "Buildup Factor" (B) into the equation to yield:

$$I = I_o B e^{-(\mu/\rho)(\rho x)} \qquad \text{Equation 11-3}$$

Dose Buildup Factors have been tabulated for a variety of beam types, absorbing materials and photon energies. An example tabulation is given in Exhibit 11-2.

The determination of the correct value for B in a specific situation entails the following steps:

Step 1: Evaluate the value of $(\mu/\rho)(\rho x)$ for the situation.

Step 2: Refer to the Buildup Factors table that pertains to the beam type, absorbing material and gamma energy of interest. (Sometimes interpolating B values between given values of $(\mu/\rho)(\rho x)$ is helpful in increasing the accuracy of the B estimate).

Exhibit 11-2.
Buildup Factors

Isotropic Source μx $[(\mu/\rho)(\rho x)]$

Shielding Material	Energy [MeV]	4	7	10
Fe	0.5	1.98	3.09	5.98
	1	1.87	2.89	5.39
	2	1.76	2.43	4.13
Pb	0.5	1.69	2	2.27
	1	2.26	3.02	3.74
	2	2.51	3.66	4.84

Monodirectional Source

Water	0.5	9.05	20	35.9
	1	6.27	11.5	18
	2	4.28	6.96	9.87

Step 3: Plug B value into Equation 11-3.

Step 4: Solve the equation for the unknown.

Problem 11-5

What thickness of Pb would reduce the exposure rate from a point Cs-137 source photons (E = 0.662 MeV) from 1 R/hr to 5 mR/hr, including Buildup Factor?

Step 1: Isolate the unknown variable:

$$\frac{I}{I_0} = e^{-(\mu/\rho)(\rho x)}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{5x10^{-3}}{1} = e^{-(0.125)(11.3)(x)}$$

$\ln (5 \text{ x} 10^{-3}) = -(0.125)(11.3)(x)$

$-5.3 = -(0.125)(11.3)(x)$

$x = 3.75$ cm

$(\mu/\rho)(\rho x) = (0.125)(11.3)(3.75) = 5.3$

$\therefore B = 1.8$ from table by interpolation

Step 5: Solve for the unknown:

$$\frac{I}{I_0 B} = e^{-(0.125)(11.3)(x)}$$

$$\frac{5x10^{-3}}{(1)(1.8)} = e^{-(0.125)(11.3)(x)}$$

$$x = \frac{5.83}{(0.125)(11.3)} = 4.13 \text{ cm}$$

Step 6: Conduct reality check:

Thicknesses are of the same range for small buildup factors.

Problem 11-6

What is the ratio of the transmitted to incident photon dose rate for monodirectional 500 keV photons and 50 cm water shield, including Buildup Factor?

Step 1: Isolate the unknown variable:

$(\mu/\rho)(\rho x) = (0.119)(1)(50) = 5.95$

Step 2: Simplify the equation:

The equation is already simplified.

Step3: Validate the problem setup:

Ratio is unitless

Step 4: Plug in known quantities:

B = 16 from table by interpolation.

$$\frac{I}{I_0} = 16\ e^{-(0.119)(1)(50)}$$

Step 5: Solve for the unknown:

$$\frac{I}{I_0} = 4.17 \text{ x } 10^{-2}$$

Step 6: Conduct reality check:

Check to see that exponentials are unitless.

Beta Attenuation

It is not often recognized that WITHIN THEIR RANGE, beta particles are also attenuated exponentially, that is to say that for x<< range:

$$I = I_o e^{-(\mu/\rho)(\rho x)} \qquad \text{Equation 11-4}$$

where,

$\mu/\rho = E_{\text{max}}^{-1.43}$ [cm^2/gm]

E_{max} = maximum energy of betas [MeV]

Problem 11-7

What fraction of incident 2 MeV betas would be expected to penetrate an Al shield of 100 mg/cm^2 thickness?

Step 1: Isolate the unknown variable:

$$\frac{I}{I_0} = e^{-(\mu/\rho)(\rho x)} \qquad \mu/\rho = E_{\text{max}}^{-1.43} = 2^{-1.43} = 0.371\text{cm}^2/\text{g}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{I}{I_0} = e^{-(0.371)(0.1)}$$

Step 5: Solve for the unknown:

$$\frac{I}{I_0} = 0.964$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-8

What flux of 1 MeV betas penetrates a water shield of 2.5 mm thickness if 10^3 betas/cm^2/sec are incident on the shield?

Step 1: Isolate the unknown variable:

$$I = I_0\, e^{-(\mu/\rho)(\rho x)} \qquad \mu/\rho = E_{max}^{-1.43} = 1^{-1.43} = 1$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

betas/cm^2/sec = betas/cm^2/sec

Step 4: Plug in known quantities:

$$I = 10^3\, e^{-(1)(1)(0.25)}$$

Step 5: Solve for the unknown:

$$I = 10^3 \bullet 0.779 = 779 \text{ betas/cm}^2\text{/sec}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-9

What dose rate reduction would be expected on a field of 4 MeV betas with an Al shield of 0.5 cm thickness?

Step 1: Isolate the unknown variable:

$$\frac{I}{I_0} = e^{-(\mu/\rho)(\rho x)} \qquad \mu/\rho = E_{max}^{-1.43} = 4^{-1.43} = 0.138 \qquad \rho_{Al} = 2.7 \text{ g/cm}^3$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{I}{I_0} = e^{-(0.138)(2.7)(0.5)}$$

Step 5: Solve for the unknown:

$$\frac{I}{I_0} = 0.830 \Rightarrow 17\% \text{ reduction}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Neutron Absorption

Neutrons are also attenuated exponentially. The formulation that is most commonly used for this situation is:

$$\Phi = \Phi_o e^{-\Sigma_t x} \qquad \text{Equation 11-5}$$

where,

Φ = transmitted neutron flux or dose rate

Φ_o = incident neutron flux or dose rate

Σ_t = σN

where,

σ = interaction cross section [cm^2]

N = number of atoms/cm^3

x = shield thickness [cm]

Since neutron interaction processes are complex and very dependent on neutron energy, sometimes it is necessary to insert a Buildup Factor variable in this equation as we did for gammas. A discussion of neutron buildup factor quantification is beyond the scope of this book.

For point neutron sources, an equation that includes flux reduction via attenuation and with distance from the source is:

$$\Phi = \frac{\Phi_o e^{-\Sigma_R x}}{r^2} \qquad \text{Equation 11-6}$$

where,

Σ_R = total removal cross section [cm^{-1}]

x = thickness of absorbing material [cm]

r = distance from source to point of interest [cm]

Problem 11-10

What is the ratio of output to input flux of neutrons from an iron shield of 10 cm thickness, if its attenuation cross-section is $10^{-23}cm^2$ (10 barns)?

Step 1: Isolate the unknown variable:

$$\frac{\phi}{\phi_0} = e^{-\Sigma_t x} \qquad \Sigma_t = \sigma N = 10^{-23} \bullet \frac{6.02x10^{23}}{55.8} \bullet 7.86 = 0.848 \text{ cm}^{-1}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

Ratio is unitless.

Step 4: Plug in known quantities:

$$\frac{\phi}{\phi_0} = e^{-(0.848)(10)}$$

Step 5: Solve for the unknown:

$$\frac{\phi}{\phi_0} = 2.08 \text{ x } 10^{-4}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 11-11

What is the flux at a distance of 1 meter from a neutron source with ϕ_0 at 1 cm = 10^8 n/cm^2/sec that is stored at the center of a 50 cm diameter container of water, if the total removal cross section is 0.1 cm^{-1}?

Step 1: Isolate the unknown variable:

$$\phi = \phi_0 \frac{e^{-\Sigma_R x}}{r^2}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

n/cm^2/sec = n/cm^2/sec

Step 4: Plug in known quantities:

$$\phi = \frac{10^8 e^{-(0.1)(25)}}{100^2}$$

Step 5: Solve for the unknown:

$\phi = 821$ n/cm^2/sec

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Neutron Activation

Under certain combinations of neutron energy and absorbing material, the stable absorbing material can be made radioactive (activated). It is often of interest how much radioactive material has been produced by specified neutron irradiation conditions. This type of evaluation is achieved by use of the following equation:

$$A_{t2} = K\sigma\phi n(1 - e^{-\lambda t_1})e^{-\lambda t_2} \qquad \text{Equation 11-7}$$

where,

A = measured activity at time t_2 [ncpm]

t_2 = time increment between end of irradiation and the time at which target is counted

K = efficiency of counter

σ = activation cross section [cm^2/atom/neutron]

ϕ = neutron flux [neutrons/cm^2/sec]

n = total number of target nuclei (not to be confused with 'n' used to denote the neutron itself)

λ = decay constant of radioactive material [1/time]

t_1 = irradiation time

e = base of natural logarithm

With activation cross sections being readily available from the Chart of the Nuclides, activation problems are solved just as radioactive decay problems were in the last chapter.

Problem 11-12

What is the highest activity in [dps] that can result from an irradiation of 10 grams of Na-23 for 100 hours with a thermal neutron flux of 10^{10} n/cm^2/sec if the activation cross section is 0.4 barns?

Step 1: Isolate the unknown variable:

$$A_{t2} = K\sigma\phi n(1-e^{-\lambda t})\,e^{-\lambda t_2} \qquad n = \frac{6.02x10^{23} \bullet 10}{23} = 2.62 \text{ x } 10^{23}$$

Step 2: Simplify the equation:

$$A_{t2}/K = \sigma\phi n(1-e^{-\lambda t})e^{-\lambda t_2}$$

Step 3: Validate the problem setup:

$$\frac{c}{\sec} = \left(\frac{c}{dis}\right)\left(\frac{n}{cm^2 \sec}\right)(atoms) = \frac{c}{\sec}$$

Note: N x atoms = dis

Step 4: Plug in known quantities:

$$\frac{A_{t2}}{K} = (0.4 \times 10^{-24})(10^{10})(2.62 \times 10^{23})(1-e^{-\frac{0.693 \bullet 100}{15}})$$

Step 5: Solve for the unknown:

$$\frac{A_{t2}}{K} = 1.05 \times 10^{9} \bullet 0.99 = 1.04 \times 10^{9} \text{ disintegrations}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Problem 11-13

What irradiation time does it take for the reaction in problem 11-12 to reach a practical maximum?

Step 1: Isolate the unknown variable:

≈7 half-lives for equilibrium

Step 2: Simplify the equation:

Half-life = 15 hours

Step 3: Validate the problem setup:

Hours = hours

Step 4: Plug in known quantities:

Step 5: Solve for the unknown:

Time for equilibrium = 7 • 15 ≈ 100 hours

Step 6: Conduct reality check:

Check arithmetic.

Problem 11-14

What would be the highest activity that could be induced in the sample in problem 11-12?

Step 1: Isolate the unknown variable:

$$\left(\frac{A_{t2}}{K}\right)_{\max} = \sigma\phi n$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

(1/sec • unitless) = (cm^2/atom • neutron) • (neutron/cm^2 • sec) • atoms

1/sec = 1/sec

Step 4: Plug in known quantities:

$$\left(\frac{A_{t2}}{K}\right)_{\max} = 1.05 \times 10^{9} \quad \text{from problem 11-12}$$

Step 5: Solve for the unknown:

See step 4.

Step 6: Conduct reality check:

Check arithmetic.

Problem 11-15

What would be the total count at 10% efficiency in problem 11-12 after 2 hours post irradiation?

Step 1: Isolate the unknown variable:

$$A_{t2} = K \bullet 1.04 \times 10^{9} \bullet e^{-\frac{0.693 \bullet 2}{15}}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

1/sec = 1/sec

Step 4: Plug in known quantities:

$$A_{t2} = (0.1)(1.04 \times 10^{9})(0.912)$$

Step 5: Solve for the unknown:

$$A_{t2} = 9.48 \times 10^{7} \text{ counts}$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

CHAPTER 12

INTERNAL DOSIMETRY

A later chapter will discuss external dosimetry, assessing the radiation exposure received from penetrating radiation sources outside the body. This chapter discusses radiation exposure resulting from radioactive material deposited inside the body. Both external and internal dosimetry must be considered together when operating a radiation protection program. Limits on the radiation dose allowed to a worker or member of the public are in terms of total effective dose equivalent, that is the sum of internal and external dose. This chapter discusses this and other related concepts.

Basic Dosimetry Concepts

Radiation dose, or absorbed dose, is the measure of the amount of energy imparted to matter from ionizing radiation. The special unit for absorbed dose is the rad. One rad is equal to 100 ergs of energy deposited per gram of matter. Absorbed dose can be calculated regardless of the type of ionizing radiation (gamma radiation, beta radiation, or alpha radiation) or the type of matter (air, water, body tissue). The System International (SI) unit for absorbed dose is the Gray. One Gray is equal to one Joule of energy imparted to one kilogram of matter. One Gray equals 100 rad.

For radiation protection, we are more interested in the amount of energy absorbed in a mass of body tissue and the type of radiation involved. An absorbed dose of one rad to a person means that 100 ergs of energy were deposited by ionizing radiation per gram of body tissue. However, different types of ionizing radiation have different potential to cause damage in the body. In other words, one rad of absorbed dose from gamma radiation causes different damage compared to one rad of absorbed dose from alpha radiation. The radiation protection community developed an expression that attempts to place radiation exposure of the body on even terms as it relates to the potential for causing damage to the body. The concept that was developed is dose equivalent. The special unit for dose equivalent is the rem. The SI unit is the Sievert. One Sievert is equal to 100 rem. Dose equivalent (H) is the product of the absorbed dose (D), a quality factor (QF), and a distribution factor (DF).

$$H = D \bullet QF \bullet DF \qquad \text{Equation 12-1}$$

The quality factor is used to represent the relative difference in damaging potential for different kinds of ionizing radiation. For alpha particles, the quality factor is 20. For beta, gamma, and x-rays, the quality factor is 1. For neutrons, the quality factor is a function of the neutron energy. The factor is 2 for thermal neutrons and 10 for fast neutrons. In other words, the absorbed dose from alpha radiation causes 20 times more damage than the same absorbed dose for beta radiation.

The distribution factor is rarely used in radiation protection calculations. It applies to non-uniform deposition of certain bone seeking radionuclides, such as radium. Note that the dose equivalent is a defined concept to be used for radiation protection. The quality factors represent a typical value for the relative effectiveness for a type of radiation to cause damage and does not necessarily represent measured values. The only measured value in the dose equivalent concept is absorbed dose.

Problem 12-1

What is the dose equivalent, in rem, delivered from an absorbed dose of 47 millirads from beta particles? 47 millirads from alpha particles? What are the dose equivalents, in Sieverts, from these absorbed doses?

Step 1: Isolate the unknown variable:

$$H = D \bullet QF \bullet DF$$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem set-up:

H (rem or Sieverts) = D (rad or Gray) • QF (rem/rad or Sievert/Gray) • DF (unitless)✓

The dose equivalent is Sieverts is equal to the product of the dose equivalent in rem and 0.01 Sievert/rem.

Step 4: Plug in known quantities:

Beta absorbed dose (QF = 1, DF = 1)

$$H\ (\text{rem}) = 47 \times 10^{-3}\ (\text{rad}) \bullet 1\ (\text{rem/rad}) \bullet 1\ (\text{unitless})$$

Alpha absorbed dose (QF = 20, DF = 1)

$$H\ (\text{rem}) = 47 \times 10^{-3}\ (\text{rad}) \bullet 20\ (\text{rem/rad}) \bullet 1\ (\text{unitless})$$

Step 5: Solve for the unknown:

Beta absorbed dose (QF = 1, DF=1)

$$H = 47 \times 10^{-3}\ \text{rem} = 47 \times 10^{-5}\ \text{Sievert}$$

Alpha absorbed dose (QF = 20, DF=1)

$$H = 94 \times 10^{-2}\ \text{rem} = 94 \times 10^{-4}\ \text{Sievert}$$

Step 6: Conduct reality check:

> For absorbed doses from beta and gamma/x-rays, the dose equivalent is equal to the asorbed dose (the QF is one). For alpha particles, the dose equivalent is twenty times the absorbed dose. ✓

Internally-Deposited Radioactive Material

Radioactive material can be taken into the body by a variety of means. These means include inhalation, ingestion, absorption through the skin, and injection (either inadvertent or for medical reasons). Once in the body, a radionuclide will be incorporated into body tissues. Different radionuclides will be incorporated into different organs and eliminated from the body at different rates. How effectively a specific radionuclide is incorporated into the body depends on the chemical solubility of the element. Relatively insoluble elements are not absorbed into the blood stream and are readily excreted from the body. Relatively soluble elements are readily absorbed into the body.

While in the body, the radioactive material will decay and the radiation emitted during this decay will deposit energy into body tissue. For non-penetrating radiation (alpha radiation, most beta radiation, and some very low energy gamma rays), the energy of the radiation is imparted into the tissue containing the radioactive material. For example, uranium taken into the body is accumulated in the kidney. The alpha particles emitted from the uranium deposits their energy in the kidney. In another example, iodine is concentrated in the thyroid. The beta particles emitted from radioisotopes of iodine deposit their energy in the thyroid. For penetrating radiation, gamma rays and some high-energy beta particles, the radioactive material in one tissue will irradiate other parts of the body. For example, cesium is accumulated in body muscle. The beta particle emitted from cesium-137 deposits its energy into the muscle tissue. The 0.662 MeV gamma ray emitted from the barium-137m daughter product (which is in secular equilibrium with the cesium-137) will irradiate all organs of the body.

Radioactive material is eliminated from the body in two ways: radioactive decay or elimination through body excreta. Each individual radionuclide has a radioactive half-life, the time it takes for half of the atoms of a given radionuclide to decay. Elements also have a half-life in the body, a biological half-life. For example, tritium has a biological half-life of 10 days compared to its radiological half-life of 12.3 years. Since both biological elimination and radioactive decay are occurring, a radionuclide will have an effective half-life in the body. The effective decay constant is the sum of the radiological and biological decay constants:

$$\lambda_E = \lambda_R + \lambda_B$$ Equation 12-2

$$\lambda_E = \frac{\ln 2}{T_{1/2E}}$$ Equation 12-3

$$\lambda_R = \frac{\ln 2}{T_{1/2R}}$$

$$\lambda_B = \frac{\ln 2}{T_{1/2B}}$$

Problem 12-2

What is the effective half-life of tritium in the body? What is the effective half-life of gold-199 in the body?

$T_{1/2B}$ (hydrogen-3) = 12 days

$T_{1/2R}$ (hydrogen-3) = 12.3 years

$T_{1/2B}$ (gold-199) = 3 days

$T_{1/2R}$ (gold-199) = 3.15 days

Step 1: Isolate the unknown variable:

$$\lambda_E = \lambda_R + \lambda_B$$

$$\lambda_R = \frac{\ln 2}{T_{1/2R}}$$

$$\lambda_B = \frac{\ln 2}{T_{1/2B}}$$

$$T_{1/2E} = \frac{\ln 2}{\lambda_E}$$

Step 2: Simplify the equation:

$$\lambda_E = \frac{\ln 2}{T_{1/2R}} + \frac{\ln 2}{T_{1/2B}}$$

$$T_{1/2E} = \frac{\ln 2}{\lambda_E}$$

Step 3: Validate the problem set-up:

$$\lambda_E(days^{-1}) = \frac{\ln 2}{T_{1/2R}(days)} + \frac{\ln 2}{T_{1/2B}(days)} \checkmark$$

$$T_{1/2E}(days) \quad = \quad \frac{\ln 2}{\lambda_E(days^{-1})} \checkmark$$

Step 4: Plug in known quantities:

Tritium

$$\lambda_E(days^{-1}) = \frac{\ln 2}{12.3 \bullet 365\ (days)} + \frac{\ln 2}{12\ (days)} = 0.058(days^{-1})$$

$$T_{1/2E}(days) \quad = \quad \frac{\ln 2}{0.058(days^{-1})}$$

Gold-199

$$\lambda_E(days^{-1}) = \frac{\ln 2}{3.15\ (days)} + \frac{\ln 2}{3\ (days)} = 0.45(days^{-1})$$

$$T_{1/2E}(days) \quad = \quad \frac{\ln 2}{0.45(days^{-1})}$$

Step 5: Solve for the unknown:

Tritium

$$T_{1/2E}(days) \quad = \quad \frac{\ln 2}{0.058(days^{-1})} = 12(days)$$

Gold-199

$$T_{1/2E}(days) \quad = \quad \frac{\ln 2}{0.45(days^{-1})} = 1.5(days)$$

Step 6: Conduct reality check:

For radionuclides like tritium, where the radiological half-life and biological half-life are very different, the effective half-life is virtually equal to the shorter of the two half-lives (one half-life dominates). For radionuclides like Au-199, where the two half-lives are similar, the effective half-life is something less than either half-life (both elimination processes act to reduce the amount of material in the body).✓

Committed Dose Equivalent and Effective Dose Equivalent

Since different radionuclides will remain in the body for different amounts of time, the effects of two intakes of different radionuclides can be vastly different. For the time that a radionuclide (or its radioactive daughter products) is in a body, the body is absorbing dose and is receiving a dose equivalent. To account for this exposure to radiation over time, a concept has been developed. The Committed Dose Equivalent is the dose equivalent received from an intake of radioactive material over a set time period. Typical time periods include 50 years and 70 years. The 50-year time period is used to represent a worker's lifetime and the 70-year time period represents the life span of a member of the public.

There is still another radiation protection concept developed to assess the impacts of internally deposited radioactive material. As mentioned earlier, different elements will go to different organs of the body, *e.g.*, iodine to the thyroid, uranium to the kidneys, and cesium to the muscle. Also, radioactive material that is inhaled will deposit some of its radiation in the lungs, depending on how easily the material is taken into the bloodstream from the lungs. Different organs exhibit different sensitivities to radiation. The same dose equivalent delivered to different organs can result in different risks. The concept is called Effective Dose Equivalent.

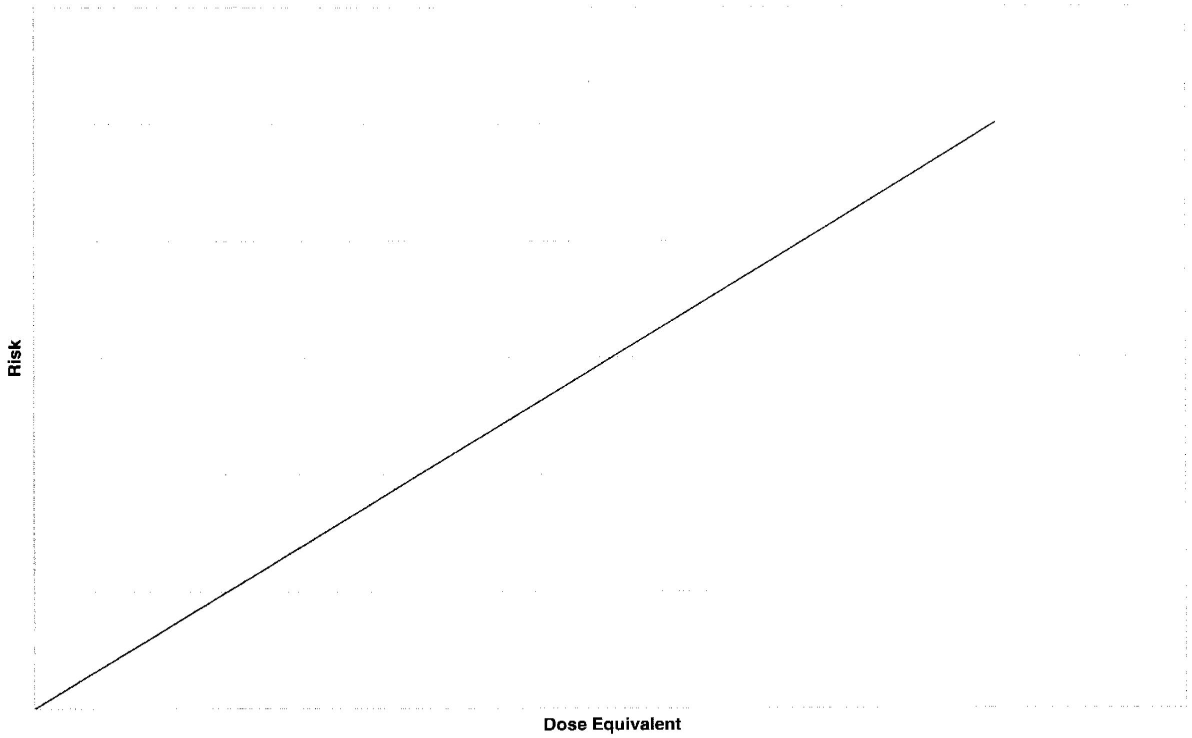
To understand the concept of Effective Dose Equivalent (EDE), we must consider that for exposure to radiation at levels typical for workers and the public, effects on the body are stochastic. In other words, there is a probability of an impact on the body associated with a given exposure. It is not certain that the effect will occur, just that there is a probability of an effect. For radiation exposure, the effects considered in radiation protection are fatal cancer and genetic effects passed to future generations.

The stochastic effects are modeled as a linear relationship between risk of an effect and dose equivalent without a threshold. This model means that for any dose, no matter how small, there is an associated risk; and since the relationship is linear, if you double the dose, you double the risk. Exhibit 12-1 illustrates the relationship.

The concept of Effective Dose Equivalent, H_E, uses the relative risk of inducing a fatal cancer or genetic effect from an exposure to a specific organ to weight the dose equivalents received by that organ. That is:

$$H_E = \sum_{all\ organs} w_t \bullet H_t \qquad \text{Equation 12-4}$$

Exhibit 12-1.
Linear Non-Threshold Theory



where,

H_t = dose equivalent deliver to organ (tissue) t
w_t = weighting factor for organ (tissue) t

The weighting factors are given in Exhibit 12-2.

As with dose equivalent, internally deposited radioactive material can result in an effective dose equivalent over a period of time. Therefore, the effective dose equivalent delivered from the intake of radioactive material over a set period of time is the committed effective dose equivalent (CEDE).

Exhibit 12-2.
Organ Weighting Factors

Organ	Weighting Factor
Gonads[1]	0.25
Breast	0.15
Red Bone Marrow	0.12
Lung	0.12
Thyroid	0.03
Bone Surface	0.03
Remainder[2]	0.30

[1] Represents genetic risk to first 2 generations.

[2] Weighting factor of 0.06 for the five organs with the next highest dose equivalent.

To determine the dose equivalent, effective dose equivalent, and committed effective dose equivalent, you must consider all the types of radiation emitted from a radionuclide, all the organs that contain radioactive material, and all the possible target organs that could have energy deposited in them from the "source" organs. This determination can be a laborious process. Many radionuclides emit many gamma rays along with either beta particles or alpha particles. Also, many elements go to more than one organ in the body, meaning that there can be many source organs. Fortunately, the calculations have been done for us. Large databases exist that contain dose conversion factors. These conversion factors estimate the collective dose equivalent and CEDE per unit intake of material.

A comprehensive database was published by the International Commission on Radiation Protection (ICRP), in their Publication 30, *Limits for Intakes of Radionuclides by Workers*. ICRP-30 estimates the 50-year committed dose equivalent and CEDE for most of the radionuclides of interest in the nuclear industry for ingestion and inhalation of radionuclides in a variety of chemical forms. Another database is DOE/EH-071, *Internal Dose Conversion Factors for Calculation of Dose to the Public.* This publication is used by the DOE in estimating CEDE for both DOE workers and the public surrounding DOE sites.

Problem 12-3

What is the 50-year committed effective dose equivalent, in rem, delivered from inhaling 1 microcurie of strontium-90 (D class).

Step 1: Isolate the unknown variable:

$$H_E = \sum_{\text{all organs}} w_t \bullet H_t$$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem set-up:

$$H_E(rem) \;=\; \sum_{all\ organs} w_t(unitless) \bullet H_t(rem) \checkmark$$

Step 4: Plug in known quantities:

The only data we have is the weighting factors. To determine the organ dose equivalent values, we consult a reference that provides these unit dose equivalent values. One example of an appropriate reference is DOE/EH-0071, *Internal Dose Conversion Factors for Calculation of Dose to the Public*, July 1988, U.S. Department of Energy.

This reference gives us an organ dose of 1.2 rem per microcurie to the red bone marrow (weighting factor 0.12) and 2.7 rem per microcurie to the bone surfaces (weighting factor 0.03).

$$H_E = 1.2 \text{ rem} \bullet 0.12 + 2.7 \text{ rem} \bullet 0.03$$

Step 5: Solve for the unknown:

$$H_E = 0.23 \text{ rem}$$

Step 6: Conduct reality check:

DOE/EH-0071, *Internal Dose Conversion Factors for Calculation of Dose to the Public*, also presents the CEDE per unit intake (microcurie). The reference gives a CEDE of 0.23 rem/μCi.✓

Dose Equivalent Limits

The concepts of dose equivalent, committed dose equivalent, and effective dose equivalent (and committed effective dose equivalent) are used to verify that exposure of workers and the public are within established dose equivalent limits.

In the United States, dose equivalent limits are established by either the U.S. Nuclear Regulatory Commission (NRC) or the U.S. Department of Energy (DOE), depending on who has authority over the operations related to the radioactive material. Both organizations have codified their dose limits, *i.e.,* made them into law. The NRC requirements are found in Title 10, Code of Federal Regulations, Part 20 (10 CFR Part 20). The NRC as given certain states, termed Agreement States, the authority to act in the NRC's behalf for activity associated with radioactive material in that state. Agreement States have dose limits that are verbatim to 10 CFR Part 20. The DOE requirements are in 10 CFR Part 834 (for the public) and 10 CFR Part 835 (for workers).

The occupational dose limits established by the NRC and DOE are identical and include:

> (1) A total effective dose equivalent of 5 rems (0.05 sievert); (2) The sum of the deep dose equivalent for external exposures and the committed dose equivalent to any organ or tissue other than the lens of the eye of 50 rems (0.5 sievert); (3) A lens of the eye dose equivalent of 15 rems (0.15 sievert); and (4) A shallow dose equivalent of 50 rems (0.5 sievert) to the skin or to any extremity. (b) All occupational exposure received during the current year shall be included when demonstrating compliance with § 835.202(a). (c) Exposures from background, therapeutic and diagnostic medical radiation, and voluntary participation in medical research programs shall not be included in dose records or in the assessment of compliance with the occupational exposure limits.

Likewise, the NRC and DOE have limited the dose that a member of the general public living near a nuclear facility can receive. This limit is 100 millirem per year. The U.S. Environmental Protection Agency has established a dose equivalent limit of 25 millirem per year from non-reactor fuel cycle facilities and limits the dose equivalent from airborne effluents to 10 millirem per year through their National Environmental Standards for Hazardous Air Pollutants (NESHAPs) regulations.

When we look at the dose limits for workers presented above, a few new concepts must be discussed. First, a worker is limited to 5 rem per year total effective dose equivalent (TEDE). The TEDE is the sum of the dose equivalent from external radiation and the CEDE for internally deposited radioactive material. Also, the rules limit the dose equivalent (not effective dose equivalent) to a single organ to 50 rem (15 to the lens of the eye). This additional limit is to protect the workers from non-stochastic effects of radiation.

Non-stochastic effects are those effects that exhibit a threshold dose and the severity increases with increasing dose above that threshold. Note that these effects differ from stochastic effects, which have no threshold and the chance of incurring an incident increases with increasing dose. An example of a non-stochastic effect of radiation is cataracts of the eye. Exposure to radiation to the lens of the eye will cause a cataract. The threshold dose for a cataract ranges from 200 to 500 rem. Below this dose, no effects will be seen. As the dose increases above the threshold, the cataract becomes more severe.

The dose limits established above are legal limits. To ensure that these limits are not exceeded, many nuclear facilities establish administrative limits. The administrative limits would establish exposure limits less than those in the regulations. For example, a facility might establish an administrative limit for the TEDE of 1 rem per year. If an individual exceeds this administrative limit, but not the regulation limit of 5 rem, then the facility has not broken the law. However, unless planned, this exceeding the administrative limit might reflect a breakdown in the radiation safety program.

Annual Limit on Intake and Derived Air Concentration

To control exposure to internally deposited radioactive material, the CEDE limits have been translated into a convenient term: Annual Limit on Intake (ALI). As the name suggests, these limits establish the maximum quantity of a radionuclide that can be taken into the body that would result in a CEDE or committed dose equivalent to an organ exceeding the NRC or DOE limit. The ALI is established based on a 5 rem CEDE limit (stochastic) or a 50 rem dose equivalent limit to a single organ (non-stochastic), whichever yields a more restrictive ALI.

The ALI is given by the following formula:

$$ALI\,(stochastic) \quad = \quad \frac{5\ rem}{\sum w_t \bullet H_t\ rem/\mu Ci} \qquad \text{Equation 12-5}$$

$$ALI\,(non-stochastic) \quad = \quad \frac{50\ rem}{H_{t,\max.organ}\ rem/\mu Ci} \qquad \text{Equation 12-6}$$

The ALI is defined for each radionuclide and is based on a dose per unit intake factor. Since one ALI delivers a dose equal to the annual limit, a worker who intakes a mixture of radionuclides can receive only a fraction of an ALI for each of the radionuclides in that mixture to ensure that the 5 rem CEDE or 50 rem single organ limit is not exceeded. The "sum-of-the-fractions" rule is used to demonstrate that the limits are not exceeded. This rule states that the sum of the ratios of the annual intake for a radionuclide to that radionuclide's ALI must be less than unity to not exceed the dose limit. The mathematical expression is:

$$\sum_{overall\ radionuclides\ in\ the\ mixture} \frac{Annual\ in\,take\ of\ radionuclide\ i}{ALI\ \ for\ radionuclide\ i} \leq 1 \qquad \text{Equation 12-7}$$

Note that if the above sum-of-the-fractions is equal to one, then the worker has gotten all 5 rem of the limit from the CEDE. If the worker received any external radiation exposure, then the TEDE would be greater than the limit. To ensure that the TEDE is not exceeded, an administrative control limit could be used to reduce the ALI for the radionuclides of concern at a facility.

For worker protection from airborne radioactive material, the ALI is used to establish routine airborne concentrations, called Derived Air Concentrations (DACs). A DAC is that concentration of a radionuclide in air that would lead to an intake of one ALI over the course of 2000 hours of work (and therefore a dose of 5 rem CEDE or 50 rem to a single organ). The DAC is based on a worker that breathes 0.02 cubic meters of air per minute. Over 2000 hours (120,000 minutes) in a work year, the worker would breathe 2400 cubic meters of air. The DAC is the ratio of the ALI to the volume of air breathed in a year:

$$DAC = \frac{ALI\ \mu Ci}{2400\ m^3}$$ Equation 12-8

Problem 12-4

What is the inhalation ALI for 1 microcurie of strontium-90 (D class). What is the DAC?

Step 1: Isolate the unknown variable:

$$ALI\,(stochastic) = \frac{5\ rem}{\sum w_t \bullet H_t\ rem/\mu Ci}$$

$$ALI\,(non-stochastic) = \frac{50\ rem}{H_{t,\max.organ}\ rem/\mu Ci}$$

$$DAC = \frac{ALI\ \mu Ci}{2400\ m^3}$$

Step 2: Simplify the equation:

The equation is in a simplified form.

Step 3: Validate the problem set-up:

$$ALI\,(\mu Ci) = \frac{5\ rem}{\sum w_t \bullet H_t\ rem/\mu Ci} \checkmark$$

$$ALI\,(\mu Ci) = \frac{50\ rem}{H_{t,\max.organ}\ rem/\mu Ci} \checkmark$$

$$DAC(\mu Ci/m^3) = \frac{ALI\ \mu Ci}{2400\ m^3}$$

Step 4: Plug in known quantities:

From Problem 12-3, we know that the CEDE is 0.23 rem/μCi and that the maximum organ dose equivalent is 2.7 rem to the bone surfaces.

$$ALI(stochastic, \mu Ci) = \frac{5\ rem}{0.23\ rem/\mu Ci}$$

$$ALI(non-stochastic, \mu Ci) = \frac{50\ rem}{2.7\ rem/\mu Ci}$$

$$DAC(\mu Ci/m^3) = \frac{ALI\ \mu Ci}{2400\ m^3}$$

Step 5: Solve for the unknown:

$$ALI(stochastic, \mu Ci) = 22\ \mu Ci$$

$$ALI(non-stochastic, \mu Ci) = 19\ \mu Ci$$

$$DAC(\mu Ci/m^3) = \frac{19\ \mu Ci}{2400\ m^3} = 0.0079\,\mu Ci/m^3$$

Step 6: Conduct reality check:

The non-stochastic ALI is the most restrictive. This occurs when the maximum organ dose is to an organ with a small weighting factor and there are few organs that contribute to the CEDE.✓

Determining Internal Dose: Air Monitoring and Bioassay

Although not the only route of intake of radioactive material by a worker, inhalation is the dominant route. The regulations allow for one of two ways to determine the amount of internally deposited radioactive material accumulated by a worker. One way is through the analysis of air samples. The other way is to assay the amount of material in the body.

As discussed above, the Derived Air Concentration is used to determine the CEDE delivered to a worker in a year. Air monitors would be set up in a workplace. These monitors would be capable of providing a representative breathing zone sample. The sample would be analyzed to determine the concentration of radionuclides in the air. Logs or similar methods would be used to determine the amount of time the worker was exposed to a given concentration. The product of the exposure time and the concentration, divided by 2000 hours must be kept below the DAC. For a mixture of radionuclides, the sum-of-the-fractions rule is used:

$$\frac{Exposure\ time \bullet Concentration}{2000\ hours} \leq DAC \qquad \text{Equation 12-9}$$

$$\sum_{\text{overall radionuclides in the mixture}} \frac{\textit{Exposure time} \bullet \textit{Concentration for radionuclide } i}{2000 \textit{ hours} \bullet \textit{DAC for radionuclide } i} \leq 1$$

Equation 12-10

Bioassay is another means to determine the amount of radioactive material taken into the body. There are to categories of bioassay: in *vivo* and *in vitro*. *In vivo* bioassay uses radiation detectors to count the radioactive material in the body. Whole body counters use a series of detectors to count the radiation that is emitted from the body by radioactive material in the body. Specialized systems are designed to focus on specific areas of the body, such as a chest counter, which is used to quantify the amount of material deposited in the lungs. These specialized systems are generally optimized to see radionuclides that are difficult to measure with conventional whole body counters.

Some radionuclides, including tritium, uranium, and plutonium isotopes, are difficult or impossible to quantify using whole body counters. For these radionuclides, *in vitro* bioassay is used. Body excreta, generally urine, are analyzed using radiochemical techniques. These analyses estimate the amount of material being eliminated from the body. Knowing this elimination rate and the type of exposure (routine or one-time), models are used to extrapolate the amount of material in the body and the amount originally taken into the body.

CHAPTER 13

ATMOSPHERIC DISPERSION

One of the most important considerations in responding appropriately to atmospheric releases of radionuclides, especially large accidental releases, is the dispersion of the release in the atmosphere. Much research has been done on this subject and most Emergency Operation Centers now have the applicable models, meteorological and demographic data preprogrammed for instantaneous use. It is essential to the result interpretation phase of the decision-making process, that the foundation of the processes being modeled is understood by the interpreter, who is often a radiation protection technologist.

Several sets of semi-empirical equations have been developed to describe the dispersion behavior of contaminants in turbulent atmospheres. One of the most common sets is named after one of its developers, Sutton. This set of equations quantifies the ground-level concentration of the released contaminant as a function of atmospheric stability, wind velocity and height of release. The relationships of the types of atmospheric stability with observable weather conditions are shown in Exhibit 13-1.

Sutton's equation is:

$$\overline{X} = \frac{2Q}{\pi C^2 \mu x^{2-n}} e^{-\frac{y^2+h^2}{C^2 x^{2-n}}} \qquad \text{Equation 13-1}$$

where,

$\overline{X}$ = volumetric concentration of the contaminant [mCi/m^3]

Q = emission rate [mCi/sec]

x,y = coordinates of point of interest from the foot of the stack

μ = mean wind speed [m/sec]

C = virtual diffusion coefficients in lateral and vertical directions

n = dimensionless parameter determined by atmospheric stability

h = effective stack height [m]

The values of some of these variables for the stability classes listed in Exhibit 13-1 are shown in Exhibit 13-2.

Exhibit 13-1.
Relationship of Stability Classes to Weather Conditions

A - Extremely Unstable
B - Moderately Unstable
C - Slightly unstable
D - Neutral
E - Slightly Stable
F - Moderately Stable

Exhibit 13-2.
Stability Class Characteristics

		Stack Height [m]			
Stability Class	N	25	50	75	100
A,B	0.2	0.043	0.03	0.024	0.015
C,D	0.25	0.014	0.01	0.008	0.005
E	0.33	0.006	0.004	0.003	0.002
F	0.5	0.004	0.003	0.002	0.001

Most often this equation is plotted as a curve relating X/Q vs. distance. For ground level releases the resulting curve is shown in Exhibit 13-3, and for an elevated release in Exhibit 13-4. It should be noted that elevating a release both (1) moves the maximum ground-level concentration per unit release further away from the release point and (2) reduces the magnitude of the maximum ground level concentration.

In many cases, radiological releases are made from a stack at temperatures and velocities above ambient. Both of these conditions increase the effective height of the stack through which the release is taking place. This increase in effective stack height is quantified by means of Equation 13-2:

$$h = h_a + d\left(\frac{v}{\mu}\right)^{1.4}\left(1 + \frac{\Delta T}{T}\right) \qquad \text{Equation 13-2}$$

EXHIBIT 13-3
GROUND LEVEL RELEASE

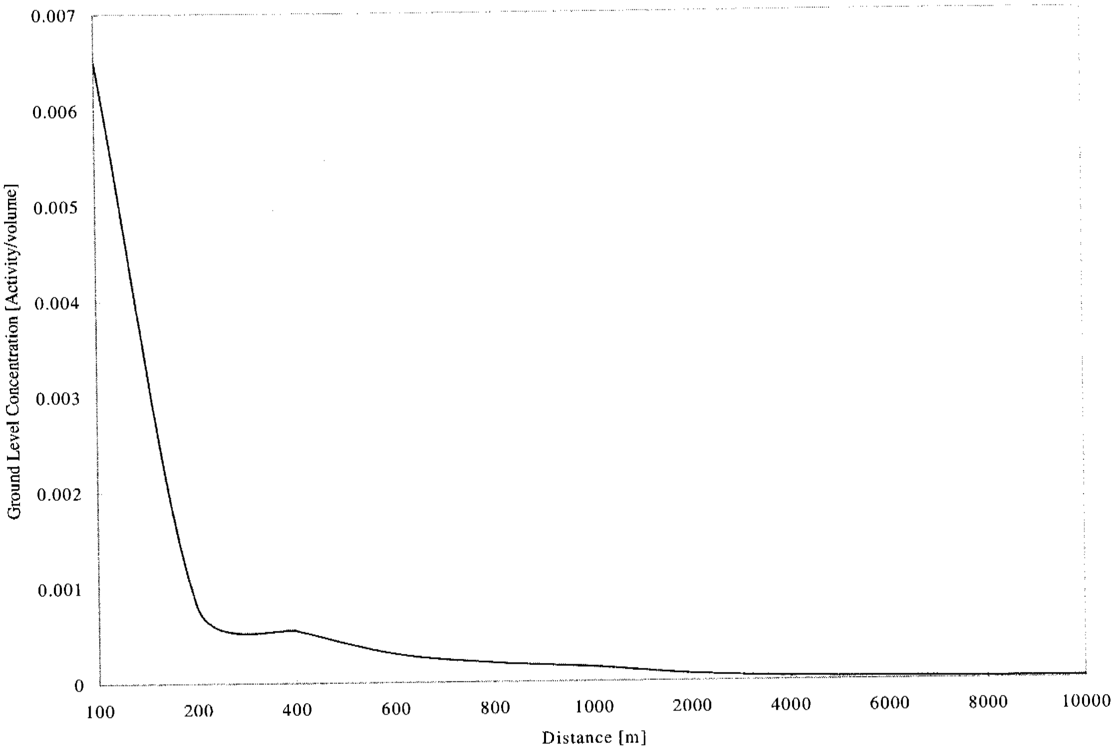
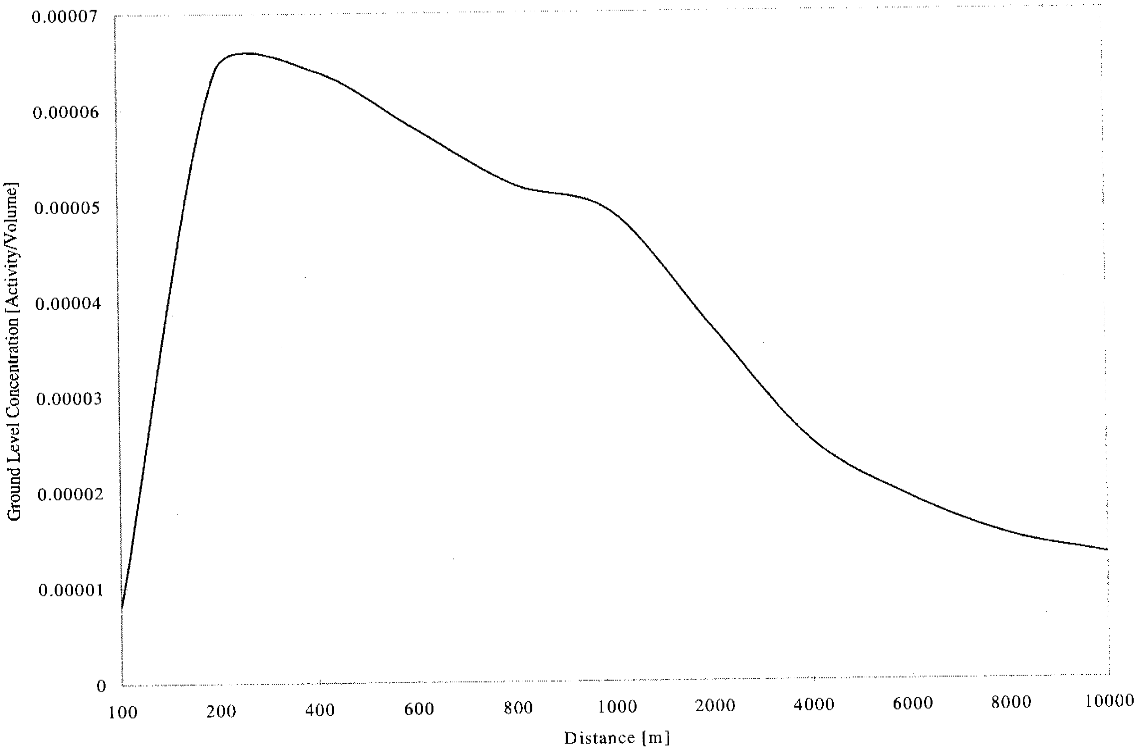


EXHIBIT 13-4
ELEVATED RELEASE



where,

h = effective stack height [m]

h_a = actual stack height [m]

d = diameter of stack outlet [m]

v = exit velocity of gas [m/sec]

μ = mean wind speed [m/sec]

ΔT = difference between ambient and gas temperature [°C]

T = absolute temperature of exit gas [°K] (0°C = 273°K)

It is always interesting, and often important, to know where the maximum X/Q is relative to the distance from the release point, and what the maximum X/Q value is at that point. These quantities are calculated using the following equations:

$$x_m = \left(\frac{h^2}{C^2}\right)^{\frac{1}{2-n}} \qquad \text{Equation 13-3}$$

$$\overline{X}_{\max} = \frac{2Q}{\pi \mu h^2 e} \qquad \text{Equation 13-4}$$

It is also instructive to know the minimum time of arrival of the release to the point of maximum concentration or to a destination of demographic importance. For short half lived radionuclides these transit times will affect the proportion of the released inventory that will ultimately arrive at any point of interest. These times are calculated by using the equation:

$$t = \frac{d}{\mu} \qquad \text{Equation 13-5}$$

where,

t = transit time of contaminant [sec]

d = distance to point of interest [m]

μ = mean wind speed [m/sec]

Problem 13-1

The Ar-41 effluent from an air-cooled reactor is 1000 Ci/sec. The effluent is discharged through a stack 75 meters high, with a speed of 2 m/sec and a stable atmosphere. What is the ground level concentration of Ar-41 at 2 km from the stack on the centerline of the plume?

Step 1: Isolate the unknown variable:

$$\overline{\mathrm{X}} = \frac{2Q}{\pi C^2 \mu x^{2-n}} (\mathrm{e}^{-\frac{y^2+h^2}{C^2 x^{2-n}}})$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{mCi}{m^3} = \frac{mCi}{\sec} \bullet \frac{\sec}{m} \bullet \frac{1}{m^2}$$

Step 4: Plug in known quantities:

$$\overline{\mathrm{X}} = \frac{2x10^3}{\pi(8x10^{-3})(2)(5.98x10^5)} \mathrm{e}^{-\frac{(75)^2}{(8x10^{-3})(2x10^3)^{2-0.25}}}$$

Step 5: Solve for the unknown:

$$\overline{\mathrm{X}} = 6.65 \text{ x } 10^{-2} \bullet 3.09 \text{ x } 10^{-1} = 2.05 \text{ x } 10^{-2} \text{ Ci/m}^3$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 13-2

If the Ar-41 release in the previous problem were at a temperature of 87°C, ambient temperature were 27°C, the effluent velocity were 10 m/sec and the diameter of the stack were 2 m, what would be its ground level concentration 2 km from the stack?

Step 1: Isolate the unknown variable:

$$\overline{\mathrm{X}} = \frac{2Q}{\pi C^2 \mu x^{2-n}} (\mathrm{e}^{-\frac{y^2+h^2}{C^2 x^{2-n}}}) \qquad \mathrm{h} = \mathrm{h}_a + \mathrm{d}(\frac{v}{\mu})^{1.4}(1 + \frac{\Delta T}{T})$$

$$\mathrm{h} = 75 + 2(\frac{10}{2})^{1.4}(1 + \frac{60}{300})$$

$$\mathrm{h} = 75 + (19)(1.2) = 97.8 \text{ m}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{mCi}{m^3} = \frac{mCi}{\sec} \bullet \frac{\sec}{m} \bullet \frac{1}{m^2}$$

Step 4: Plug in known quantities:

$$\overline{\mathrm{X}} = \frac{2x10^3}{\pi(5x10^3)(2)(5.98x10^5)} \mathrm{e}^{-\frac{(98.7)^2}{(5x10^{-3})(5.98x10^5)}}$$

Step 5: Solve for the unknown:

$$\overline{\mathrm{X}} = (1.06 \text{ x } 10^{-1})(4.08 \text{ x } 10^{-2}) = 4.32 \text{ x } 10^{-3} \text{ Ci/m}^3$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 13-3

In problem 13-2 where does the maximum X occur and what is $\mathrm{X}_{\max}$?

Step 1: Isolate the unknown variable:

$$\mathrm{x}_{\max} = (\frac{h^2}{C^2})^{\frac{1}{2-n}}$$

$$X_{\text{max}} = \frac{2Q}{\pi\mu h^2 e}$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

$$\text{m} = (\text{m}^2)^{1/2}$$

$$\frac{mCi}{m^3} = \frac{mCi}{\sec} \bullet \frac{\sec}{m} \bullet \frac{1}{m^2}$$

Step 4: Plug in known quantities:

$$\text{x}_{\text{max}} = \left(\frac{(97.8)^2}{(5x10^{-3})}\right)^{-\frac{1}{2-0.25}} = (1.91 \text{ x } 10^6)^{0.571}$$

$$X_{\text{max}} = \frac{2x10^3}{\pi(2)(97.8)^2(2.72)}$$

Step 5: Solve for the unknown:

$$\text{x}_{\text{max}} = 3.86 \text{ x } 10^3 \text{m}$$

$$X_{\text{max}} = 1.22 \text{ x } 10^{-2} \text{ Ci/m}^3$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

Problem 13-4

How long does it take the released Ar-41 to reach the maximum X distance in problem 13-3? What proportion of the released Ar-41 decays before reaching this distance?

Step 1: Isolate the unknown variable:

$$\text{t} = \frac{d}{\mu}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\sec = \frac{m}{\frac{m}{\sec}}$$

Step 4: Plug in known quantities:

$$t = \frac{3.86x10^3}{2}$$

Step 5: Solve for the unknown:

$t = 1.93 \text{ x } 10^3 \sec = 32.2$ hours

$$\frac{A}{A_0} = e^{-\frac{0.693t}{\lambda}} \qquad = e^{-\frac{0.693 \bullet 32.2}{1.83}} = 5\ 06 \text{ x } 10^{-6}$$

Step 6: Conduct reality check:

Check to see that exponentials are unitless.

If the released contamination is a particle and settles to the ground, not behaving as a gas in transit, as is characterized in Equations 13-1 - 13-5, $\overline{X}$ is calculated, accounting for increased ground deposition of the particles during transit, using the equation:

$$\overline{X} = \frac{Q}{\pi C^2 \mu x^{2-n}} e^{-\frac{y^2+(z')^2}{C^2 x^{2-n}}} \qquad \text{Equation 13-6}$$

where,

$$z' = h - \frac{x v_t}{\mu}$$

where,

v_t = terminal settling velocity [m/sec]

z' = elevation of the plume [m]

and the other variables are as defined in Equation 13-1

Problem 13-5

If the Ar-41 in problem 13-1 were a particulate radionuclide with a terminal settling velocity of 1 cm/sec and were released at the same rate, under the same conditions, what would be X at 2 km distance?

Step 1: Isolate the unknown variable:

$$X = \frac{Q}{\pi C^2 \mu x^{2-n}} e^{-\frac{y^2+(z')^2}{C^2 x^{2-n}}} \qquad z' = h - \frac{x v_t}{\mu} = \frac{98.7 - (2x10^3 \bullet 10^{-2})}{2}$$

$$z' = 39.4 \text{ m}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{mCi}{m^3} = \frac{mCi}{\sec} \bullet \frac{\sec}{m} \bullet \frac{1}{m^2}$$

Step 4: Plug in known quantities:

$$X = \frac{10^3}{\pi (5x10^{-3})(2)(5.98x10^5)} e^{-\frac{(39.4)^2}{(5x10^{-3})(5.98x10^5)}}$$

Step 5: Solve for the unknown:

$$X = (5.32 \text{ x } 10^{-2})(5.95 \text{ x } 10^{-1}) = 3.17 \text{ x } 10^{-2} \text{ Ci/m}^3$$

Step 6: Conduct reality check:

Check to see that exponential is unitless.

CHAPTER 14

STATISTICS

A health physicist employs statistics to manage data collected as part of their day-to-day activities. Statistics is used to make an inference or observation about a set of data. This set of data is often called a *population* when describing statistical characteristics and the term population will be used throughout this chapter. The mathematical field of statistics can be broken into two groups, one descriptive statistics and the other inferential statistics. In descriptive statistics, the data you deal with is generally the complete population and you summarize the data for descriptive purposes. In inferential statistics, you draw conclusions about a large population from a subset of samples. The health physicist encounters both of these aspects of statistics. For example, a population may be 50 drums of contaminated soil and the contact dose rate for each of those drums is known. Additionally, soil samples have been taken from 10 of those drums. Descriptive statistics is used to summarize the dose rate information, such as the average dose rate of the drums, the maximum dose rate of the drums and the distribution of dose rates. Inferential statistics is used to estimate the characteristics of all 50 drums from the soil analysis results of the 10 samples, such as average concentration and standard deviation. This chapter discusses characteristics used in both descriptive and inferential statistics.

Data Presentation

A common component of both descriptive and inferential statistics is a data set, or population of data. As described above, in descriptive statistics, often the entire population of data is known, while in inferential statistics, we are attempting to infer characteristics about a large population of data from a subset of data. Regardless of the case, you must first organize your data. The most common methods for organizing data are tabular, putting the data in a table, or graphical. Exhibit 14-1 below presents the number of points scored by a professional basketball team in their first 27 games and whether they won (W) or lost (L) the game. Exhibit 14-1 represents just one possible tabulation of the data. Other possibilities include separate tables for wins and losses or listing the data from most points to fewest points scored. How data are tabulated is affected by how the data will ultimately be used.

Exhibit 14-2 provides a graphical representation of the data. In this figure, the points per game are plotted and the data points connected. This graphing technique allows you to see trends in the data. This type of plot is useful when presenting data that represents a population of data at a single point over time. The graph will give you a quick indication if the values are increasing, decreasing, or remaining constant. One example is the plot of survey results of contamination levels in a room over a time period. The plot can tell you if the levels are remaining constant or changing and may trigger actions depending on the trend (*e.g.*, decontaminate the area, upgrade the protective clothing requirements).

Exhibit 14-1
Points Scored by Basketball Team

Game #	Points	W or L?	Game #	Points	W or L?
1	104	W	15	105	W
2	101	W	16	107	W
3	99	W	17	98	W
4	132	W	18	84	L
5	118	W	19	99	L
6	109	W	20	92	L
7	113	W	21	119	W
8	121	W	22	119	W
9	97	W	23	109	W
10	118	W	24	83	L
11	119	W	25	98	W
12	86	L	26	109	W
13	118	W	27	94	W
14	95	L			

Exhibit 14-3 presents a different representation. In this figure, the numbers of points are divided into bins, each representing a spread of nine or ten points. Then, the numbers of games where points were scored in that bin are indicated. This type of graphical presentation is called a histogram. A histogram is used to show the distribution of data over the range of values. Exhibit 14-3 shows that for most games, the team scores between 93 and 122 points. A common distribution is a normal distribution. The classic example of this distribution is the bell curve for grades. Exhibit 14-4 presents a normal distribution.

Numerical Descriptions of Data - Tendency

Graphical and tabular data presentations provide some insight to populations. There are also numerical descriptions for data populations. These descriptions summarize the data in terms of its tendency and variability. Measures of tendency include mean, median, and mode and are discussed below. Variability is measured in terms of variance and standard deviation and is discussed in a later section.

The mean is the most common numerical description. The mean is the arithmetic average of the values in a data set. Each data set has only one mean. The mean is calculated by the following:

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

Exhibit 14-2.
Line Plot of Points per Game

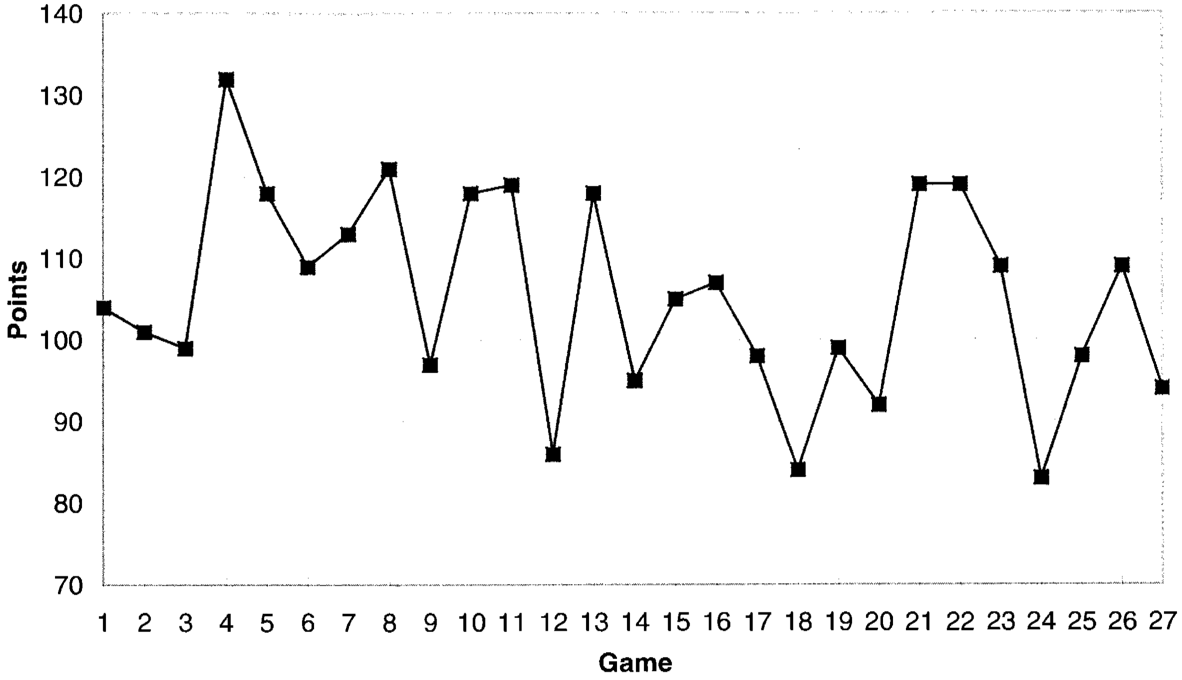


Exhibit 14-3.
Histogram of Points per Game

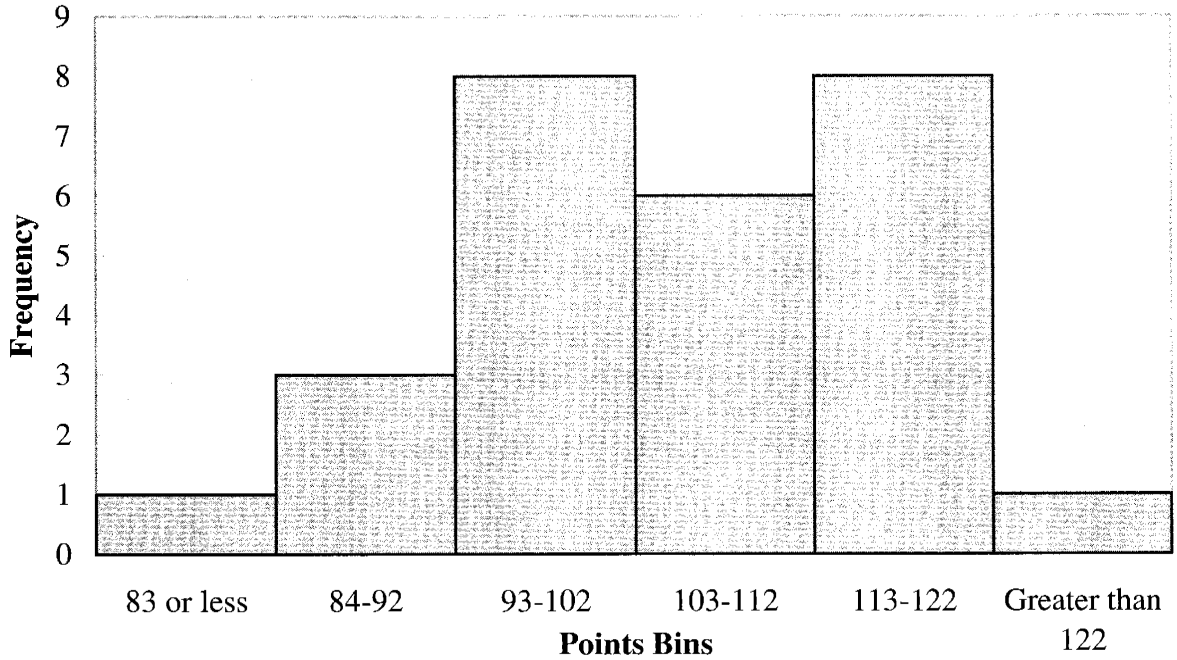
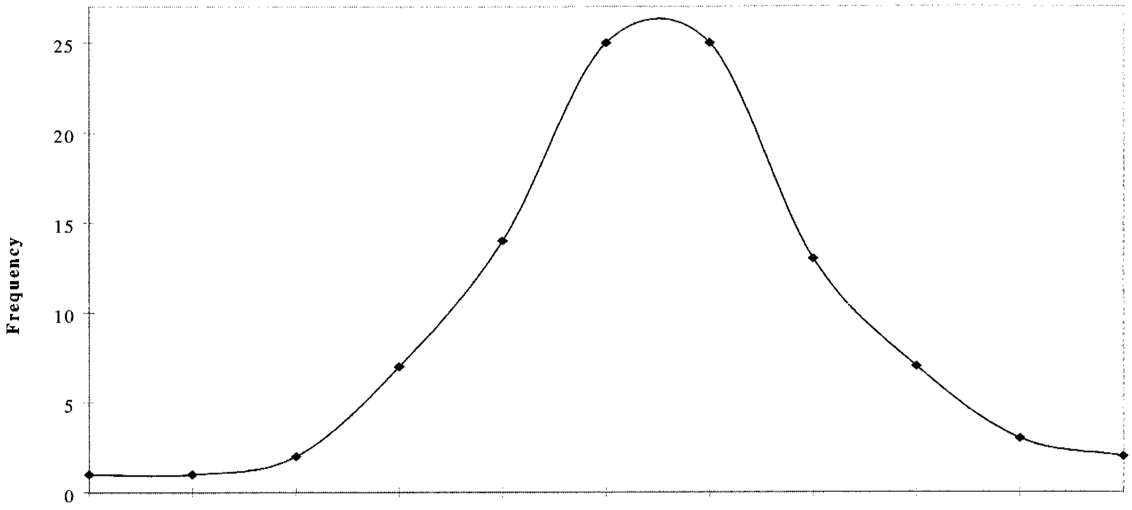


Exhibit 14-4.
Normal Distribution



where,

μ = arithmetic mean of a set of values, $x_1, x_2, x_3, \ldots x_n$,
n = number of values in the set

In our data set presented in Exhibit 14-1, the team averaged 105.4 points per game over the 27 games presented in the data. In other words, the mean of the population presented in that Exhibit is 105.4 points. Although a data set has only one mean, a population can be subdivided and means determined for the subsets. For example, our basketball team averaged 109.9 points per win and 89.8 points per loss over the 27 games.

The mean of a population is affected by extreme values. The value of the mean can change greatly if extreme data points, or outliers, are added or excluded. The smaller your data population, the more it is affected by outliers. For example, if our basketball team scores 132 points in Game 28, then its average increases a point per game from 105.4 to 106.4 points. Compare this increase to the increase after the team scored 132 in Game 4. The team's average increased almost eight points per game, from 101.3 to 109 points.

The mean from subsets can be combined to determine the mean for the larger population. The following expression is used to combine means. Caution: You can not simply sum the means for each subset and divide by the number of subsets. The means must be "weighted" by the number of data points in the subset ("n" and "m" in the equation that follows).

$$\mu = \frac{(\sum_{i=1}^{n} x_i \bullet n) \quad + \quad (\sum_{j=1}^{m} x_j \bullet m)}{n + m}$$

For example, [(109.9 points per win) * (21 wins) + (89.8 points per loss) * (6 losses)] / 27 games = 105.4 points per game, our average for the entire population.

The mean can be used for descriptive or inferential statistics. In our example, the mean describes the exact average of the number of points over those 27 games. However, given an 82 game season, we might infer that the team would score 8,643 points over the course of the season. This total is the product of the average, 105.4 points per game, and the number of games, 82. Of course, our average is based on the team's performance over the *first* 27 games. Many things can happen - injuries, trades, and uneven schedules - that can impact a team's performance over the course of a season. To more accurately infer the total points scored over an 82-game season from the results of 27 games, we would need to randomly select 27 games from the population of 82 results. Of course, all 82 games would have to be played before the random subset could be selected. When selecting a subset of a population to represent the population as a whole, care must be taken. Improperly selecting a subset can bias the inferences about the population.

Why select a subset to represent the population? Usually, you select a subset of a population to represent the entire population when the entire population is too large to manage. An every-day example of this is polling data. A poll might say that 65 percent of American voters approve of a candidate for president. The pollster did not ask every voter and determined that 65 percent of them approved of the candidate. Instead, the pollster selected a representative subset of the population of voters. The pollster did have to make sure that the subset was *representative*. A subset made up entirely of college students might yield very different results compared to a subset made up entirely of members of local Veterans of Foreign War group. Similarly, if a health physicist were determining the levels of removable contamination on the floor of a room, they would sample a subset of the room. If the room is 400,000 cm^2, they might take forty 100-cm^2 smears to represent a room. As with the pollster, the health physicist would not want to bias the samples. The smears would be taken randomly throughout the room, not just in one corner of the room.

The mean is not the only measure of tendency. The median and mode of a data set provide insight into the population. The median is the central value. Fifty percent of the data values are less than the median and fifty percent are greater than the median. The mode is the most frequent or probable data value.

Used in conjunction with the mean, the median and mode help to describe the distribution of data. For a set of data whose median and mode are greater than the mean, the data distribution is skewed to the right; more data points have values greater than the mean. This type of skewed data occurs if there are outliers on the low end of the data range. For a set of data whose median and mode are less than the mean, the data distribution is skewed to the left - more data points have values less than the mean. This type of skewed data occurs if there are outliers on the high end of the data range. If the mean, median, and mode are close to the same value, then the data set resembles the bell curve discussed earlier.

Numerical Descriptions of Data - Variability

Variability provides another descriptive measure of a data set. The variability describes the range of the data set and the relative frequency of the data in that range. Common measures of variability when managing health physics data are variance and standard deviation.

Variance is a measure of the numerical difference between a data point value and the mean of the data set. The variance is expressed as follows:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}$$

where,

s^2 = variance
μ = arithmetic mean of a set of values, $x_1, x_2, x_3, \ldots x_n$,
n = number of values in the set

Standard deviation of a population or subset of data is the square root of the variance. In our example data set in that Exhibit 14-1, the variance of the data set is 158.4 and the standard deviation is 12.6. Note that the square root of 158.4 has both 12.6 and -12.6 as answers. The standard deviation is expressed as the positive square root of the variance.

The standard deviation is a convenient measure for data that are normally distributed, that is, data that exhibits a bell curve. As an approximation, 68 percent of the data in a normally distributed set of data lie within ± one standard deviation from the mean, 95 percent of the data lie within ± two standard deviations from the mean, and nearly all of the data lie within ± three standard deviations from the mean.

If we again look at the data set in Exhibit 14-1, we have a mean of 105.4 and a standard deviation of 12.6. From Exhibit 14-3, we see that the data resembles a bell curve (although it is depressed in the center). If we use the approximation from above, we would expect 68 percent of the point values to be in a range of 105.4 ± 12.6 points, or 92.8 points to 118 points. We would expect 95 percent of the point values to be in a range of 105.4 ± 25.2 points, or 80.2 points to 130.6 points. If we take 68 percent of 27 games, we get 18 games while 95 percent of 27 games is 26 games. When you examine the data in Exhibit 14-1, you see that 18 of the point values lie in the range of 92.8 points to 118 points, while 26 point values lie in the range of 80.2 points to 130.6 points.

Although it worked out exactly for our data set, note that the above rule is only an approximation. It can be used to test if a data set is normally distributed or estimate the range of a population if you know its mean and standard deviation and know that the population is normally distributed. For example, the next chapter will discuss counting statistics. Radioactive decay is a random event and the results of counting a sample are a normal distribution of values, described by a mean and standard deviation. The health physicist can use the counting result (the mean) and the standard deviation to determine a complete estimate of the activity of a sample, including a measure of the uncertainty in the activity estimate.

Summary

To summarize this overview of basic concepts of statistics lets leave the basketball court and return to our original example, our 50 drums of contaminated soil. Exhibit 14-5 presents the contact dose rates, the net cesium-137 and uranium-235 concentrations for 50 drums.

Exhibit 14-5.
Data on Soil Drums

Drum #	Dose Rate, mR/hr	Cs-137, pCi/g	U-235, pCi/g	Drum #	Dose Rate, mR/hr	Cs-137, pCi/g	U-235, pCi/g
1	9	20100	12	26	22	20600	2
2	8	18100	8	27	48	20700	3
3	11	22500	9	28	10	20800	8
4	9	19200	19	29	10	20900	7
5	10	21000	25	30	9	18900	11
6	9	19300	22	31	10	21900	13
7	19	19400	11	32	10	21000	25
8	9	19500	12	33	55	21050	0
9	9	18200	22	34	9	20100	19
10	10	21200	19	35	9	20050	13
11	9	19600	25	36	10	20150	13
12	24	19700	4	37	28	18400	2
13	72	19800	2	38	10	21400	15
14	9	19900	8	39	9	20100	11
15	9	20000	10	40	9	20050	29
16	9	19000	19	41	10	20150	22
17	10	22050	17	42	31	20200	3
18	9	20100	23	43	9	20000	6
19	10	20200	29	44	9	18500	29
20	10	20300	21	45	10	21500	29
21	37	20400	3	46	10	20180	8
22	10	20500	18	47	9	20020	15
23	9	18300	16	48	9	20100	13
24	10	21300	14	49	9	20100	8
25	10	21800	20	50	9	18800	7

What is the average contact dose rate for the drums? What are the median, variance, and standard deviation of the contact dose rates? What can you conclude about the dose rates of the drums? What are the average Cs-137 and U-235 concentrations for the drums? What are the median, variance, and standard deviation of the concentrations? What can you infer about the concentration of Cs-137 and U-235 in the drums?

Step 1: Isolate the unknown variable:

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}$$

$$s = \sqrt{s^2}$$

Step 2: Simplify the equation:

The equations are in simplified forms.

Step 3: Validate the problem set-up:

The mean and standard deviation have the same units as the population. The units for variance are the square of the units of the population. ✓

Step 4: Plug in known quantities and Step 5: Solve for the unknown:

Drum	Dose Rate, mR/hr	x-m	$(x-m)^2$
1	9	-5.46	29.81
2	8	-6.46	41.73
3	11	-3.46	11.97
4	9	-5.46	29.81
5	10	-4.46	19.89
6	9	-5.46	29.81
7	19	4.54	20.61
8	9	-5.46	29.81
9	9	-5.46	29.81
10	10	-4.46	19.89
11	9	-5.46	29.81
12	24	9.54	91.01
13	72	57.54	3310.85
14	9	-5.46	29.81
15	9	-5.46	29.81
16	9	-5.46	29.81
17	10	-4.46	19.89
18	9	-5.46	29.81
19	10	-4.46	19.89

	20	10	-4.46	19.89
	21	37	22.54	508.05
	22	10	-4.46	19.89
	23	9	-5.46	29.81
	24	10	-4.46	19.89
	25	10	-4.46	19.89
	26	22	7.54	56.85
	27	48	33.54	1124.93
	28	10	-4.46	19.89
	29	10	-4.46	19.89
	30	9	-5.46	29.81
	31	10	-4.46	19.89
	32	10	-4.46	19.89
	33	55	40.54	1643.49
	34	9	-5.46	29.81
	35	9	-5.46	29.81
	36	10	-4.46	19.89
	37	28	13.54	183.33
	38	10	-4.46	19.89
	39	9	-5.46	29.81
	40	9	-5.46	29.81
	41	10	-4.46	19.89
	42	31	16.54	273.57
	43	9	-5.46	29.81
	44	9	-5.46	29.81
	45	10	-4.46	19.89
	46	10	-4.46	19.89
	47	9	-5.46	29.81
	48	9	-5.46	29.81
	49	9	-5.46	29.81
	50	9	-5.46	29.81
Sum:		723		8260
Average:	723/50 = 14.46			
Variance:				8260/(50-1) = 168.58
Standard deviation:				12.98

The above calculation is for the dose rate of the drums. Note that you must calculate the mean for the population before you can calculate the variance and standard deviation.

It can be repeated for the concentration of cesium-137 and uranium-235 in the drums. The mean for the cesium-137 concentration is 20,142 pCi/g, the variance is 1,020,400 (pCi.g)^2 and the standard deviation is 1010 pCi.g. The mean for the uranium-235 concentration is 14 pCi/g, the variance is 67 (pCi.g)^2 and the standard deviation is 8 pCi.g.

Step 6: Conduct reality check:

The values for mean, variance, and standard deviation can be calculated using most of today's scientific calculators or with built-in functions found in spreadsheet software. ✓

CHAPTER 15

COUNTING STATISTICS

Since radioactive disintegration is a statistical process, it is not surprising that mathematical statistics are frequently applied to the data produced in counting operations. The major applications are the fundamental statistics of standard deviation and other measures of dispersion.

In analytical work, we are only able to run a small, but finite number of analyses. Statistics enable us to give a quantitative description of our small number of analytical results and to give an estimated description of the entire population (the infinite number of possible analytical results).

One desirable characteristic for describing a sampling from a population is the scatter of the results about the measured mean (average) value. The simplest measure of scatter is the range; that is the largest and smallest values obtained. The range does not show dispersion as well as other measures, but is simple and is widely used for describing a small number of analyses. In radiation protection work we deal with more sophisticated estimates of population characteristics. As the number of items in our sample increases, our estimates approach the true value of the parameter.

Counting data belong to a population where the events are discrete and appear in relatively small number compared to the number that could appear in the time available. This population is best described by the Poisson distribution. Any count is a sample from this population and is a partial description of the population.

The sample standard deviation for a Poisson distributed population is given by

$$s = \sqrt{x} \qquad \text{Equation 15-1}$$

The probability of the next evaluation of a count rate being $\sqrt{x} \pm s$ is 0.683. Other ranges of the standard deviation (s) show the following probabilities:

- 0.6745 s = 0.500 probability
- 1.000 s = 0.683
- 1.645 s = 0.90
- 1.960 s = 0.95
- 2.000 s = 0.955
- 2.580 s = 0.99
- 3.000 s = 0.9974
- 4.000 s = 0.9999

Since counting data follow the Poisson distribution,

$$s = \sqrt{N} \qquad \text{since } N = \bar{x} \qquad \text{Equation 15-2}$$

where,

N = the number of counts obtained

s = standard deviation of the count. In the same way, N/t is the count rate {R} for a determination, and

$$s_R = \frac{\sqrt{N}}{t} \qquad \text{Equation 15-3}$$

is the standard deviation of the count rate. This can also be written,

$$s_R = \sqrt{R / t} \qquad \text{Equation 15-4}$$

When the counter background is of importance, an additional step is required to calculate the standard deviation of the net count. This step is based on the fact that the variance (the square of the standard deviation) of the difference of the two independent variables is the sum of their variances. Thus,

$$s_N^2 = s_B^2 + s_G^2 \qquad \text{Equation 15-5}$$

where,

s_N = standard deviation for the net counts
s_B = standard deviation for background counts
s_G = standard deviation for the gross counts (sample + background)

It is obvious that the value of s_N must be at least $s_B \sqrt{2}$, and that the background counting rate is one limiting factor in low level counting. For equal counting times, the standard deviation for the net count rate is,

$$s_{netrate} = \frac{\sqrt{N_B + N_G}}{t} \qquad \text{Equation 15-6}$$

where,

N_B = the net background counts
N_G = the net gross counts
t = counting times for both gross and background

For unequal counting times,

$$s_{netrate} = \sqrt{\frac{R_B}{t_B} + \frac{R_G}{t_G}} \qquad \text{Equation 15-7}$$

where,

R_B = background count rate
t_B = background count time
R_G = gross count rate
t_G = gross count time

<u>Problem 15-1</u>

What is the standard deviation of the count rate that results from a count of 3600 counts in 10 minutes?

Step 1: Isolate the unknown variable:

$$s_R = \sqrt{\frac{R}{t}}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

N/A

Step 4: Plug in known quantities:

$$s_R = \sqrt{\frac{360}{10}}$$

Step 5: Solve for the unknown:

$$s_R = \sqrt{36} = 6 \text{ cpm}$$

Step 6: Conduct reality check:

Check the arithmetic.

Problem 15-2

What is the standard deviation of the net count rate that results from a sample count rate of 800 cpm for a 10 minute count and a background count rate of 1 cpm for a 1 hour count?

Step 1: Isolate the unknown variable:

$$s_{netrate} = \sqrt{\frac{R_B}{t_B} + \frac{R_G}{t_G}}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

N/A

Step 4: Plug in known quantities:

$$s_{netrate} = \sqrt{\frac{1}{60} + \frac{800}{10}}$$

Step 5: Solve for the unknown:

$s_{netrate} = 8.95$ cpm

Step 6: Conduct reality check:

Check the arithmetic.

Problem 15-3

A preliminary count of a sample resulted in an approximate count rate of 50 cpm. A subsequent background count resulted in 100 counts in 2 hours. Using a 95% confidence limit, calculate the required length of a count so that the true net count rate will be within 20% of the true sample net count rate.

Step 1: Isolate the unknown variable:

20% of 50 = 10 cpm

$2\ s_{netrate} = 10$ cpm

$s_{netrate} = 5$ cpm

$$s_{netrate} = \sqrt{\frac{R_B}{t_B} + \frac{R_G}{t_G}}$$

Step 2: Simplify the equation:

$$t_G = \frac{R_G}{s^2_{netrate} - \frac{R_B}{t_B}}$$

Step 3: Validate the problem setup:

N/A

Step 4: Plug in known quantities:

$$t_G = \frac{50}{24.99}$$

Step 5: Solve for the unknown:

$t_G = 2$ minutes

Step 6: Conduct reality check:

Check the arithmetic.

The optimum division of counting time between the gross (sample + background) and background is given by

$$\frac{t_B}{t_G} = \sqrt{\frac{R_B}{R_G}} \qquad \text{Equation 15-8}$$

Problem 15-4

What would be the optimum sample and background count times for the sample in problem 15-2 if the total time allowed for the procedure were 60 minutes?

Step 1: Isolate the unknown variable:

$$\frac{t_B}{t_G} = \sqrt{\frac{R_B}{R_G}}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

N/A

Step 4: Plug in known quantities:

$$\frac{t_B}{t_G} = \sqrt{\frac{1}{800}} = 0.04$$

$$t_B = 0.04\ t_G$$

$$t_B + t_G = 60$$

Step 5: Solve for the unknown:

$$1.04\ t_G = 60 \Rightarrow t_G = 57.7 \text{ minutes} \Rightarrow t_B = 2.3 \text{ minutes}$$

Step 6: Conduct reality check:

Check the arithmetic.

Often we are interested in knowing if the difference between a sample and background is due to actual activity in the sample, or due to statistical fluctuations of the background. The technique to be described next reports the probability that the sample count rate is a result of statistical fluctuations of the background count rate. If this probability is very small, then we assume that the sample count is not a result of statistical fluctuations, but rather is a result of activity present in the sample.

In using this test, a confidence limit is chosen. Usually, the 95% confidence limit is selected. As shown earlier in this chapter, this corresponds to approximately two standard deviations. The difference of the sample and background rates is assumed to be 0. Then the probability of obtaining the actual difference is calculated. If this probability is less than or equal to 5% we reject this assumption and assume that the sample contains activity above background.

Problem 15-5

An air sample counted for 10 minutes resulted in 600 counts. A 30 minute background count resulted in 1500 counts. At the 95% confidence limit, is there any radioactivity in the air?

Step 1: Isolate the unknown variable:

$$\mathrm{R}_{net} = \frac{600}{10} - \frac{1500}{30} = 10\mathrm{cpm}$$

$$\mathrm{s}_{net} = \sqrt{\frac{R_B}{t_B} + \frac{R_G}{t_G}} = \sqrt{\frac{60}{10} + \frac{50}{30}}$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

N/A

Step 4: Plug in known quantities:

$$\frac{R_{net}}{s_{net}} = \frac{10}{2.8} = 3.5$$

Step 5: Solve for the unknown:

$$\frac{R_{net}}{s_{net}} > 2 \quad \therefore @\ 95\% \text{ there is activity in the sample}$$

CHAPTER 16

AIR SAMPLING AND COUNTING

For a reported value of radioactive material concentration in air to be meaningful, the following factors should be considered:

- Volume of Air Sampled

Nominally,

Volume = flow rate [volume/minute] • sampling time [minutes]   Equation 16-1

If the flow rates at the beginning and the end of the sampling time are different, the average of the two is generally used:

Volume = ½ (initial flow rate + final flow rate) • sampling time   Equation 16-2

It should be remembered that if only a portion of the total filter is to be counted, this must be included in the calculations of the volume of air sampled.

- Collection Efficiency

Although obviously a function of particle sizes distribution, the particulate collection efficiency of a glass fiber filter is assumed to be 100%. If this assumption is made in the evaluation of air samples, it should be made explicit in the presentation of the results. In many cases it will not be a valid assumption, and tabulations of filter efficiencies should be consulted for the appropriate efficiencies to use in the air concentration calculations.

- Counting Efficiency

For an internal proportional counter, this is usually taken to be 50%. By making this assumption, the ncpm may be roughly equated to the pCi in the sample.

- Self Absorption in the Sample

Unless the sample is unusually heavy (10 mg/cm^2) the self-absorption can be assumed to be negligible. Any self-absorption present is in part offset by the fact that the counter efficiency may be slightly higher than 50%.

Activity Calculations

The measurement of airborne radioactivity will often require calculation in order to determine the activity concentration. This calculation may require a consideration of the buildup of activity on the collection medium. This is shown on Exhibit 16-1.

General Calculations

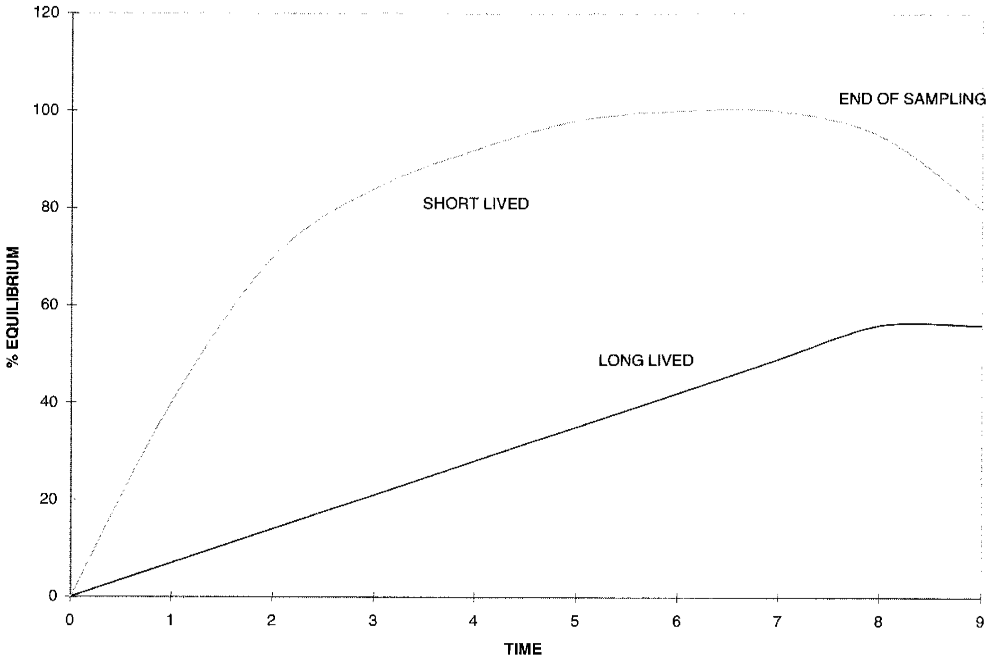
Two general considerations may be applicable to all activity concentration calculations. These are:

- sample counting
- decay time between the end of sampling and the mid-point of the counting.

The general expression for determining the quantity of a radionuclide on a collection medium is:

$$q = \frac{CR}{CE(2.22)} \qquad \text{Equation 16-3}$$

Exhibit 16-1.
Activity Relationships in Sampling



where,

q = quantity of nuclide on the collecting medium [pCi]

CR = net count rate due to the radionuclide in the sample [ncpm]

CE = counting efficiency of the counter for the specific radionuclide [ncpm/dpm]

2.22 = conversion factor [dpm/pCi]

In instances where short-lived radionuclides are being measured and if the activity at the time of sampling is to be determined, a correction for the radionuclide decay between the end of the sampling and the mid-point of counting must be made. See Exhibit 16-1. Equation 16-4 indicates this correction factor.

$$q_{corrected} = q/e^{-\lambda t} \qquad \text{Equation 16-4}$$

where,

$q_{corrected}$ = quantity of radionuclide at the end of sampling [pCi]

λ = decay constant for the radionuclide [time $^{-1}$]

t = time period between end of sampling and midpoint of counting [time]

q = quantity of radionuclide on the collecting medium

<u>Specific Calculations</u>

The activity calculations involved in the measurement of airborne radioactivity depend upon many factors. Two of the most important factors are:

- the relationship of the quantity of the radionuclide being sampled with time (constant or variable)
- the half-life of the radionuclide being sampled.

CONSTANT CONCENTRATION - LONG HALF-LIFE

When the concentration of the radionuclide in the environment is constant, and when it has a long half-life in comparison to the sampling time, the expression shown in Equation 16-5 is applicable for buildup of activity on the collection medium. The long-lived activity curve in Exhibit 16-1 depicts this case.

$$q = C \bullet F \bullet E \bullet t \qquad \text{Equation 16-5}$$

where,

q = quantity of radionuclide on the collecting medium [pCi]

C = concentration of radionuclide in the air being sampled [pCi/unit volume]

F = air flow rate through the collecting medium [volume/time]

E = collection efficiency for radionuclide under sampling conditions [fraction]

t = sampling period [time]

In this case the decay of activity on the collecting medium during sampling is of no significance due to the long half-life of the radionuclide being collected.

CONSTANT CONCENTRATION - SHORT HALF-LIFE

When the half-life of the radionuclide being sampled is less than or not too much greater than the sampling time, consideration must be given to the decay of the radionuclide on the collecting medium during sampling. It is assumed that the activity of the radionuclide in the environment is constant, i.e. environmental equilibrium exists. The sampling relationship is depicted by the short-lived activity (% equilibrium) curve in Exhibit 16-1. In this case Equation 16-6 is applicable.

$$q = \frac{C \bullet F \bullet E}{\lambda} (1 - e^{-\lambda t}) \qquad \text{Equation 16-6}$$

where,

all variables are as previously defined.

When the half-life of the radionuclide being sampled is approximately equal to or somewhat greater than the sampling time, Equation 16-6 is applicable. When the half-life is much less than the sampling time, Equation 16-6 can be reduced to:

$$q = \frac{C \bullet F \bullet E}{\lambda} \qquad \text{Equation 16-7}$$

A "rule of thumb" for deciding when equation 16-7 is applicable is when the half-life is less than or equal to 1/7 the sampling time. For a ratio of 1/7, the error is less than one per cent.

Problem 16-1

If ¼ of a filter paper from a 1-hour air sample with initial flow rate of 10 liters/min and final flow rate of 8 liters/min is counted, what air volume should be used in the concentration calculation?

Step 1: Isolate the unknown variable:

$$V = \left(\frac{R_1 + R_2}{2}\right)t$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\text{Liters} = \frac{liters}{\text{min}} \bullet \text{min}$$

Step 4: Plug in known quantities:

$$V = \left(\frac{10+8}{2}\right)60$$

Step 5: Solve for the unknown:

V = 540 liters, but since only ¼ of the filter was used V to be used in the calculation is 540 /4 = 135 liters

Step 6: Conduct reality check:

Check the arithmetic.

Problem 16-2

If a radionuclide of half-life 5 minutes is counted for 10 minutes, immediately after sampling and gives 50 ncpm on an instrument with 50% efficiency, what is the true activity in [pCi] at the end of the sampling period?

Step 1: Isolate the unknown variable:

$$q_{corrected} = \frac{CR}{CE(2.22)} / e^{-\lambda t}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\mathrm{pCi} = \frac{ncpm}{\dfrac{ncpm}{dpm} \bullet \dfrac{dpm}{pCi}}$$

Step 4: Plug in known quantities:

$$q_{corrected} = \frac{50}{0.5 \bullet 2.22} / e^{-\frac{0.693 \bullet 5}{5}}$$

Step 5: Solve for the unknown:

$$q_{corrected} = 45.0/0.5 = 90 \text{ pCi}$$

Step 6: Conduct reality check:

Check the arithmetic.

Problem 16-3

What is the airborne concentration of a radionuclide in [pCi/l] that yields a count rate of 10 dpm from a 1-hour sample, sampled at 1 l/min?

Step 1: Isolate the unknown variable:

$$\text{concentration} = \frac{A}{V}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

$$\frac{pCi}{l} = \frac{pCi}{l}$$

Step 4: Plug in known quantities:

$$\text{concentration} = \frac{10}{2.2}/60$$

Step 5: Solve for the unknown:

concentration = 0.08 pCi/l

Step 6: Conduct reality check:

Check the arithmetic.

Problem 16-4

What is the true airborne concentration of the radionuclide in problem 16-3, if at the end of the sampling period the radionuclide is only at 50% equilibrium?

Step 1: Isolate the unknown variable:

$$q_{corrected} = \frac{q}{fractionofequilibrium}$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

pCi/l = pCi/l

Step 4: Plug in known quantities:

$$q_{corrected} = \frac{0.08}{0.5}$$

Step 5: Solve for the unknown:

$q_{corrected} = 0.16$ pCi/l

Step 6: Conduct reality check:

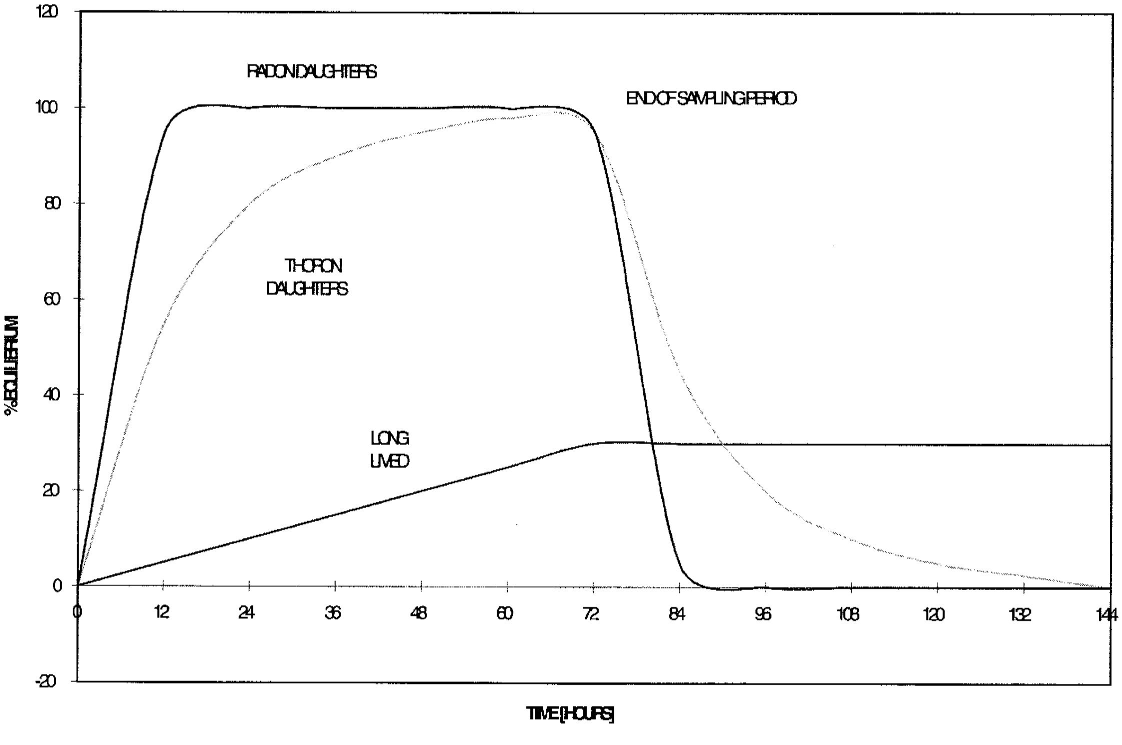
Check the arithmetic.

SERIES DECAY RADIONUCLIDES

Many airborne radionuclides exhibit series decay relationships with their daughters. In some instances the daughters are collected and analyzed, and the activity of the parent radionuclides are then calculated from those results.

As shown in Exhibit 16-1, prior to the parent and daughters reaching radioactive equilibrium the daughter activities are increasing. Exhibit 16-2 shows the buildup of alpha activity due to Radon and Thoron daughters and other long-lived alpha emitting radionuclides. Equilibrium conditions imply that the radioactivities of all members of the chain are equal, as shown for radon and thoron daughters in Exhibit 16-2. The increasing activity of the daughter being collected may have to be taken into account in the activity calculations if the sampling is done during a period prior to the attainment of radioactive equilibrium. When possible, it is desirable to sample at equilibrium conditions. This can be used to our advantage in cases like when we are sampling for radon daughters only; where a sampling period of only a few hours is desirable because then the interference from thoron activity will be small and may be neglected. Under equilibrium conditions the previously discussed equations are applicable.

Exhibit 16-2.
Alpha Activity On Air Sample



Many times a need arises for a simple and more rapid method for the determination of gross long-lived beta radioactivity in air. One such method is described below. It affords one of the simplest means of disentangling, within a reasonable length of time, the activity due to natural radionuclides from that contributed by the artificial radionuclides.

The heart of this method is the selection of the time at which the sample is to be counted. The natural activity of the air is largely a mixture of Pb-214 and Pb-212. The former has a half-life of 26.8 minutes and accounts for at least 90% of the normal air activity. The latter radionuclide has a half-life of 10.6 hours.

In this method, a period of time equal to 8 half lives of the 214 isotope is permitted to elapse before the sample is counted for the first time. This ensures that at least 99.5% of this component will have decayed. The 10.6-hour component may not be dispatched so easily, for even 7 half-lives would delay obtaining the results for three days. Instead, a calculation based upon two counts separated by approximately 24 hours is applied to remove the fraction of the total count due to the 212 isotope.

The total cpm (C_1) at time (t_1) is the sum of the cpm due to the long-lived activity (C_{LL}) and the cpm remaining from the 212 isotope collected ($C_0 e^{-\lambda t}$):

$$C_1 = C_o e^{-\lambda t_1} + C_{LL} \qquad \text{Equation 16-8}$$

Similarly

$$C_2 = C_o e^{-\lambda t_2} + C_{LL} \qquad \text{Equation 16-9}$$

Solving these equations for C_{LL} the expression becomes:

$$C_{LL} = \frac{C_2 - C_1 e^{-\lambda \Delta t}}{1 - e^{-\lambda \Delta t}} \qquad \text{Equation 16-10}$$

where,

Δt = the time interval between the two counts C_1 and C_2. This is the perfectly general form of the expression and may be applied to any time interval. However, if a specific Δt is agreed upon, a further simplification may be made. For, and only for, t_1 = 4 hours and t_2 = 28 hours:

$$C_{LL} = 1.26\,(C_c - 0.21C_1) \qquad \text{Equation 16-11}$$

Thus a sample taken at mid-morning will be counted first at mid-afternoon and the final results will be available upon making the second count at the same hour the next afternoon.

Problem 16-5

What is the count rate of the long-lived component of an alpha air sample contaminated with radon and thoron daughters, if the 4 hour count rate is 400 ncpm and the 28 hour count rate is 100 ncpm?

Step 1: Isolate the unknown variable:

$$C_{LL} = 1.26(C_2 - 0.21C_1)$$

Step 2: Simplify the equation:

The equation is already simplified.

Step 3: Validate the problem setup:

cpm = cpm

Step 4: Plug in known quantities:

$$C_{LL} = 1.26(100 - 0.21 \bullet 400)$$

Step 5: Solve for the unknown:

$$C_{LL} = 20.16 \text{ ncpm}$$

Step 6: Conduct reality check:

Check the arithmetic.

CHAPTER 17

TRIGONOMETRY

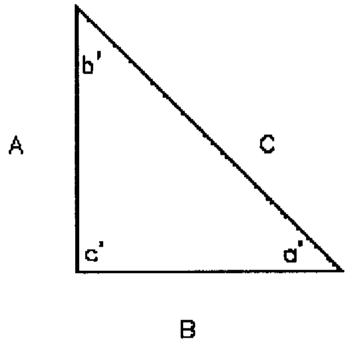
Trigonometry is the study of angles and the relationship between angles and the lines that form them. Trigonometry is used in Health Physics to solve problems related to external dosimetry, counting geometry and gamma radiation interactions. For example, you can use the basics of trigonometry to determine the dose rate from a line source of radiation, such as a pipe carrying radioactive liquid.

The fundamental concepts of trigonometry are built around the triangle. As shown in Exhibit 17-1 below, a triangle is a plane figure formed using straight line segments (*AB*, *BC*, *CA*) to connect three points (*A*, *B*, *C*) that are not in a straight line. Some mathematical characteristics of a triangle include:

- The sum of the measure of the three angle of a triangle is 180 degrees (°)
- The sum of the length of any two sides is greater than the length of the third side
- A triangle with an angle equal to 90° is known as a right triangle

The Greek mathematician Pythagorus develop a theorem, or rule, related to right triangles. The Pythagorean theorem can be used to determine the length of on side of the right triangle when you know the length of the other two sides. The term *hypotenuse* is used to describe the side of a right triangle opposite the right angle. Given that Triangle ABC is a right triangle and the measure of angle c' is 90°, the line segment C is the hypotenuse of the triangle in Exhibit 17-1. In right triangle ABC, the two sides (other than the hypotenuse) are referred to as the *opposite* and *adjacent* sides. These two terms are used when dealing with the internal angles of the triangle. Whether the side is the "opposite" or "adjacent" side depends on the angle you are discussing. In Triangle ABC, line segment A is the opposite side of angle a' and line segment B is the adjacent side. This nomenclature is easy to remember – the "opposite" side of a given angle is the side of the triangle opposite the given angle.

Exhibit 17-1.
Triangle *ABC*



The Pythagorean theorem states that in any right triangle, the square of the length of the hypotenuse (C) equals the sum of the squares of the lengths of the other two sides (A and B):

$$C = \sqrt{A^2 + B^2} \qquad \text{Equation 17-1}$$

The above equation can be solved for any of the three sides, when the length of the other two sides is known.

Now we can use the right triangle and a basic feature of the triangle to determine the unknown values for the length of sides or the measure of the angles. Remember that we said that the sum of the angle of a right triangle is 180°. Since we are using a right triangle, and we know that one of the angles is 90°, then we know that the sum of the measure of the other two angles is 90°. Understanding this feature of the right triangle, we can solve for the unknown angles if we know the length of two sides of a right triangle and vice versa. We will use the three basic trigonometric functions: sine, cosine, and tangent.

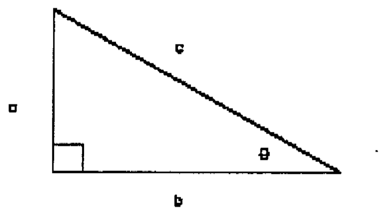
In Exhibit 17-2, side *a* is the opposite side of the angle θ and side *b* is the adjacent side of the angle θ. The terms hypotenuse, opposite side, and adjacent side are used to distinguish the relationship between an angle of a right triangle and its sides. The relationships between the length of the sides and the three trigonometric functions are:

$$Sine\ \vartheta = \frac{a}{c} = \frac{length\ of\ the\ opposite\ side}{length\ of\ the\ hypotenuse} \qquad \text{Equation 17-2a}$$

$$Cosine\ \vartheta = \frac{b}{c} = \frac{length\ of\ the\ adjacent\ side}{length\ of\ the\ hypotenuse} \qquad \text{Equation 17-2b}$$

$$Tangent\ \vartheta = \frac{a}{b} = \frac{length\ of\ the\ opposite\ side}{length\ of\ the\ adjacent\ side} \qquad \text{Equation 17-2c}$$

Exhibit 17-2.
Right Triangle

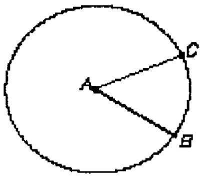


The value of any of these trigonometric functions can be determined with a calculator with trigonometric functions. To find the sine, cosine, or tangent of any angle, enter the value of the angle into the calculator and press the desired function key: SIN for sine, COS for cosine, and TAN for tangent.

In solving health physics problems, you generally know the length of the sides of the right triangle, rather than the measure of the angle and it is that measure that you are trying to determine. To solve for the measure of the angle when you know the value of the trigonometric function (in other words, the length of two of the sides of the right triangle); you use the inverse trigonometric functions. The inverse trigonometric function, also known as the *arc* function, defines the angle based on the value of the trigonometric function. For example, the sine of 30° equals 0.5; thus, the arc sine of 0.5 is 30°. The inverse of the sine is written as arcsine or $\sin^{-1}$. Either of these notations can be read as "the measure of the angle whose sin is..." The notation *arc* can be used as a prefix to any of the trigonometric functions. Note that the "-1" is not used to mean the mathematical inverse, that is 1/sine. The mathematical inverses of the sine, cosine, and tangent are the secant, cosecant, and cotangent, respectively. The "-1" means the angle whose sine is. The arc functions are usually determined with a calculator by entering the value of the function and then pressing the inverse key (INV) then the trigonometric function (for example, SIN).

The size of an angle is commonly measured in degrees. However, in some applications the size of an angle is measured in radians. A radian is defined in terms of the length of a portion of the circle (an arc) subtended by an angle at the center of a circle. An angle whose size is one radian subtends an arc whose length equals the radius of the circle. Exhibit 17-3 shows angle *BAC* whose size is one radian. The length of arc *BC* equals the radius *r* of the circle. The size of an angle, in radians, equals the length of the arc it subtends divided by the radius. One radian equals approximately 57.3 degrees. There are exactly 2 π radians in a complete revolution. Thus 2 π radians equal 360 degrees; π radians equal 180 degrees. Although the radian is defined in terms of the length of an arc, it can be used to measure any angle. Radian measure and degree measure can be converted directly. The measure of an angle in degrees is converted to radians by multiplying by π/180°. Conversely, the measure of an angle in radians is converted to degrees by multiplying by 180°/π.

Exhibit 17-3.
Radian Measure



CHAPTER 18

EXTERNAL DOSIMETRY

Up to this point in the book we have only talked of gamma dose rates and fluxes from point sources. Mastery of radiation protection problems involving point sources is extremely important, but "out in the plant" much geometry other than point sources must be addressed. At the first level of complexity, that is, where there is uniform linear, area or volumetric contamination concentrations, the handling of line, area or volume sources is not that much more difficult than what has already been demonstrated for point sources. This chapter will start with the point source, addressing it in a manner consistent with the approach that must be used for the more complex one, two and three-dimensional source geometry's.

These more advanced methods deal with external gamma dosimetry problems in terms of fluxes, then convert the calculated fluxes to either exposure or dose rates after the appropriate flux calculations have been completed. The relationships between the flux incident on the material receiving the exposure or dose of interest is given in Equations 18-1 and 18-2:

$$R\ [\text{R/hr}] = 5.08 \times 10^{-2}\ \mu_a^{air}\, I_o \qquad \text{Equation 18-1}$$

where,

R = exposure rate [R/hr]
μ_a^{air} = linear absorption coefficient [cm^{-1}]
I_o = gamma flux [$\text{MeV/cm}^2\text{/sec}$]

For the dose rate, the conversion factor is

$$R\ [\text{rads/hr}] = 5.76 \times 10^{-5}\ \left(\frac{\mu_a^{medium}}{\rho_{medium}}\right) I_o \qquad \text{Equation 18-2}$$

where,

R = dose rate [rads/hr]
μ_a^{medium} = linear absorption coefficient [cm^{-1}]
ρ_{meduim} = density of the absorbing medium [g/cm^3]
μ/ρ = mass absorption coefficient [$\text{cm}^2\text{/g}$]

Point Source

For a point source the flux at a distance is given by the equation:

$$I_o = \frac{2.96x10^9 CE}{r^2} \qquad \text{Equation 18-3}$$

where,

C = activity of the source [Ci]
E = energy of the activity [γ Mev/ disintegration]
r = distance from source to the point of interest [cm]

Problem 18-1

What is the dose rate in water at 1 meter from a 50 Ci point gamma source that emits 1 MeV/dis?

Step 1: Isolate the unknown variable:

$$R = 5.76 \text{ x } 10^{-5}(\frac{\mu}{\rho})I_0$$

$$I_0 = \frac{2.96x10^9 CE}{r^2}$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$I_0 = (2.96 \text{ x } 10^9 \bullet 50 \bullet 1)/(10^2)^2 = 1.48 \text{ x } 10^7$$

Step 5: Solve for the unknown (using μ/ρ from Exhibit 11-1):

$$R = 5.76 \text{ x } 10^{-5} \bullet 0.636 \bullet 1.48 \text{ x } 10^7 = 1.64 \text{ x } 10^3 \text{ rads/hr}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Line Source

For a line source, with its geometry relative to the numbered points of interest as depicted in Exhibit 18-1, the flux equations are:

At P_1

$$I_o = \frac{2.96x10^9 C_L E}{h}(\theta_2 - \theta_1) \qquad \text{Equation 18-4}$$

where,

C_L = linear activity concentration of source [Ci/unit length]

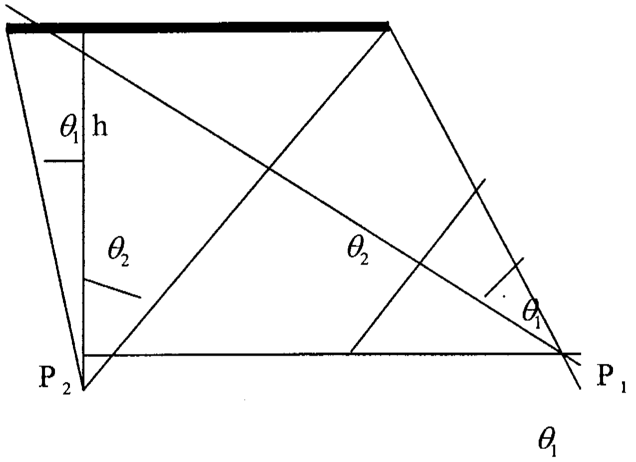
h = perpendicular distance from line to point of interest [length]

$\theta_2 - \theta_1$ = angle subtended by the line source from the perspective of the point of interest [radians] (2π radians = 360°)

At Point 2

$$I_o = \frac{2.96x10^9 C_L E}{h}(\theta_1 + \theta_2) \qquad \text{Equation 18-5}$$

Exhibit 18-1.
Line Source



where,

$\theta_1 + \theta_2$ = angle subtended by the line source from the perspective of the point of interest [radians]

Problem 18-2

What is the dose rate in air 1 meter from the middle of a nonabsorbing 4 meter long pipe filled with a 2 MeV/dis gamma emitter at a concentration of 200mCi/meter?

Step 1: Isolate the unknown variable:

$$R = 5.76 \text{ x } 10^{-5}(\frac{\mu}{\rho})\, I_0$$

$$I_0 = \frac{2.96x10^9 C_L E}{h}(\theta_1 + \theta_2)$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

Empirical equation, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$I_0 = \frac{2.96x10^9 \bullet 200 \bullet 2}{1}(2.21) = 2.62 \text{ x } 10^9$$

Step 5: Solve for the unknown:

$$R = 5.76 \text{ x } 10^{-5} \bullet 0.0445 \bullet 2.62x10^9 = 6.72 \text{ x } 10^3 \text{ rads/hr}$$

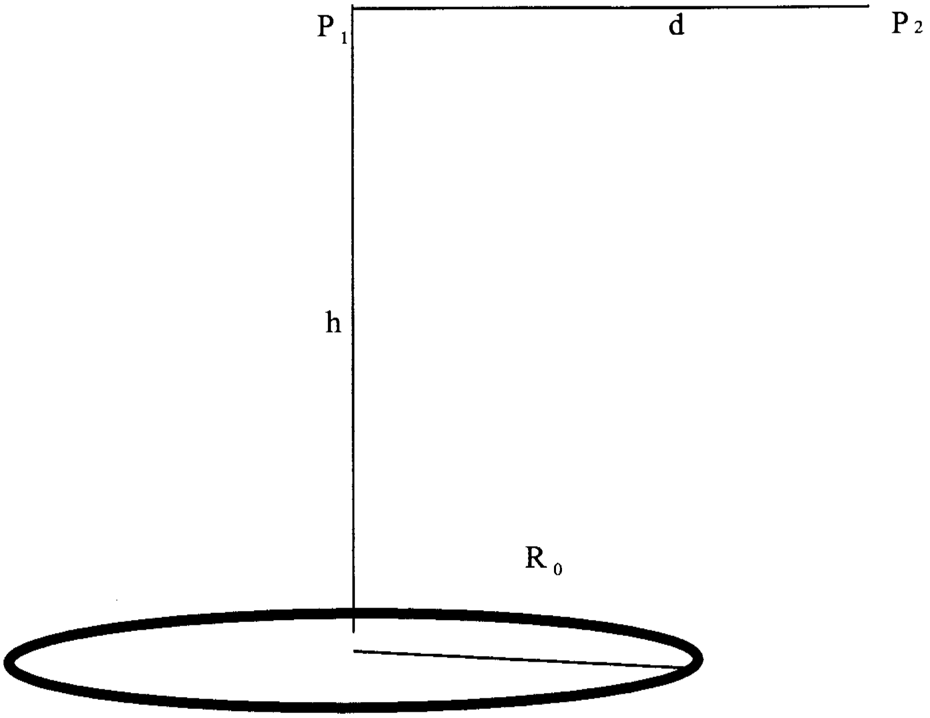
Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Disk Source

For a plane disk source of uniform activity distribution, with its geometry relative to the numbered points of interest as depicted in Exhibit 18-2, the flux equations are:

Exhibit 18-2.
Disk



At Point P_1

$$I_o = 2.96 \times 10^9 \, EC_A \pi \ln\left[\frac{(R_o^2 + h^2 - d^2) + [(R_o^2 + h^2 - d^2)^2 + 4d^2h^2]^{1/2}}{2h^2}\right] \quad \text{Equation 18-6}$$

where,

C_A = activity per unit area [Ci/cm^2]

R_o = radius of the disk source [cm]

h = perpendicular distance of the point of interest from the disk source [cm]

d = perpendicular distance from the point of interest to a point directly over the center of the disk source

At Point P_2

$$I_o = 2.96 \times 10^9 \, EC_A \pi \ln\left(\frac{R_o^2 + h^2}{h^2}\right) \quad \text{Equation 18-7}$$

Problem 18-3

What is the exposure rate 1 meter above the center of a 4 meter diameter disk source of Co-60 at a concentration of 1 Ci/m^2 ?

Step 1: Isolate the unknown variable:

$$R = 5.08 \times 10^{-2} \mu I_0$$

$$I_0 = 2.96 \times 10^9 EC_A \pi(\frac{R_0^2 + h^2}{h^2})$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

Empirical equations, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$I_0 = 2.96 \times 10^9 \bullet 2.5 \bullet 1 \bullet \pi \bullet \ln\frac{4^2 + 1^2}{1^2} = 2.32 \times 10^{10} \ln(17)$$

$$= 6.59 \times 10^{10}$$

Step 5: Solve for the unknown (using μ/ρ from Exhibit 11-1):

$$R = 5.08 \times 10^{-2} \bullet 0.0636 \bullet 1.29x10^{-3} \bullet 6.59x10^{10} = 2\ 75 \times 10^5 \text{ R/hr}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Spherical Source

For a spherical source, with uniform volumetric concentration, the flux equations are:

At the sphere's center

$$I_o = 2.96 \times 10^9 C_v E(4\pi R_s) \qquad \text{Equation 18-8}$$

where,

C_v = activity per unit volume of the source [Ci/cm^3]
R_s = radius of the spherical source [cm]

At the surface of the sphere

$$I_o = 2.96 \times 10^9 C_v E(2\pi R_s) \qquad \text{Equation 18-9}$$

Problem 18-4

What is the dose rate in air at the surface of a nonabsorbing 4 meter diameter sphere filled with Ar-41 at a concentration of 10 Ci/m^3?

Step 1: Isolate the unknown variable:

$$R = 5.76 \times 10^{-5} (\frac{\mu}{\rho}) I_0$$

$$I_0 = 2.96 \times 10^9 C_V E(2\pi R_s)$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

Empirical equations, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$I_0 = 2.96 \times 10^9 \bullet 10 \bullet 1.29 \bullet (2 \bullet \pi \bullet 2) = 4.8 \times 10^{11}$$

Step 5: Solve for the unknown (using μ/ρ from Exhibit 11-1):

$$R = 5.76 \times 10^{-5} \bullet 0.0636 \bullet 4.8x10^{11} = 1.76 \times 10^6 \text{rads/hr}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

Infinite Right Cylinder Source

For an infinite right cylinder, with uniform volumetric concentration, with its geometry relative to the points of interest as depicted in Exhibit 18-3, the flux equations are:

On the cylinder surface

$$I_o = 2.96 \times 10^9\, C_v\, E\, (4\pi R_c) \qquad \text{Equation 18-10}$$

where,

R_c = cylinder radius [cm]

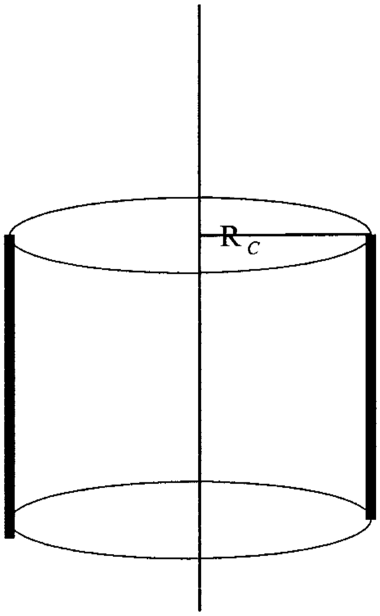
Point on cylinder axis

$$I_o = 2.96 \times 10^9\, C_v\, E(2\pi^2 R_c) \qquad \text{Equation 18-11}$$

Problem 18-5

What is the exposure rate on the surface of an infinite right cylinder of radius 1 meter filled with a 1.5 MeV/dis gamma at a concentration of 10Ci/m^3 ?

Exhibit 18-3.
Cylindrical Source



Step 1: Isolate the unknown variable:

$$R = 5.08 \text{ x } 10^{-2} \mu I_0$$

$$I_0 = 2.96 \text{ x } 10^{9} C_V E(4\pi R_c)$$

Step 2: Simplify the equation:

The equations are already simplified.

Step 3: Validate the problem setup:

Empirical equations, therefore unit analysis is not useful.

Step 4: Plug in known quantities:

$$I_0 = 2.96 \text{ x } 10^{9} \bullet 10 \bullet 1.5 \bullet (4 \bullet \pi \bullet 0.5) = 2.79 \text{ x } 10^{11}$$

Step 5: Solve for the unknown (using μ/ρ from Exhibit 11-1):

$$R = 5.08 \text{ x } 10^{-2} \bullet 0.0636 \bullet 1.29x10^{-3} \bullet 2.79x10^{11} = 1.16 \text{ x } 10^{6} \text{ R/hr}$$

Step 6: Conduct reality check:

Make independent order of magnitude estimate.

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CONVERSION FACTORS

ACTIVITY

1 terabecquerel = 1 TBq = 27 curies
1 gigabecquerel = 1 GBq = 27 millicuries
1 megabecquerel = 1 MBq = 27 microcuries
1 kilobecquerel = 1 kBq = 27 nanocuries
1 becquerel = 1 Bq = 27 picocuries

1 kilocurie = 1 kCi = 37 terabecquerels
1 curie = 1 Ci = 37 gigabecquerels
1 millicurie = 1 mCi = 37 megabecquerels
1 microcurie = 1 μCi = 37 kilobecquerels
1 nanocurie = 1 nCi = 37 becquerels

DOSE EQUIVALENT

1 sievert = 1 Sv = 100 rem
1 millisievert = 1 mSv = 100 millirem
1 microsievert = 1 μSv = 100 microrem
1 nanosievert = 1 nSv = 100 nanorem

1 kilorem = 1 krem = 10 sieverts
1 rem = 1 rem = 10 millisieverts
1 millirem = 1 mrem = 10 microsieverts
1 microrem = 1 μrem = 10 nanosieverts

ABSORBED DOSE

1 kilogray = 1 kGy = 100 krad
1 gray = 1 Gy = 100 rad
1 milligray = 1 mGy = 100 millirad
1 microgray = 1 μGy = 100 nanorad

1 kilorad = 1 krad = 10 grays
1 rad = 1 rad = 10 milligrays
1 millirad = 1 mrad = 10 micrograys
1 microrad = 1 μrad = 10 nanograys

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GREEK ALPHABET

Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta		δ
Epsilon	Ε	ϵ
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	Ξ	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	ϕ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω